Meta-analyses of partial correlations are biased: Detection and solutions

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Abstract

We demonstrate that all meta-analyses of partial correlations are biased, and yet hundreds of meta-analyses of partial correlation coefficients (PCC) are conducted each year widely across economics, business, education, psychology, and medical research. To address these biases, we offer a new weighted average, UWLS+3. UWLS+3 is the unrestricted weighted least squares weighted average that makes an adjustment to the degrees of freedom that are used to calculate partial correlations and, by doing so, renders trivial any remaining meta-analysis bias. Our simulations also reveal that these meta-analysis biases are small-sample biases ($n < 200$), and a simple correction factor of *(n-2)/(n-1)* greatly reduces these small-sample biases along with Fisher's z. In many applications where primary studies typically have hundreds or more observations, partial correlations can be meta-analyzed in standard ways with only negligible bias. However, in other fields in the social and the medical sciences that are dominated by small samples, these meta-analysis biases are easily avoidable by our proposed methods.

Keywords: partial correlation coefficients, meta-analysis, bias, small sample

Highlights

What is already known?

- All meta-analyses of partial correlation coefficients (PCCs) are biased, though the biases are relatively small in most cases.
- Hundreds of meta-analyses of PCCs are conducted each year.

What is new?

- We offer two new corrections, $UWLS_{+3}$ and RE_{ss}, that widely reduce these biases to scientific negligibility.
- Fisher's z transformations also produce small-sample biases, although they are generally negligible in application.
- $UWLS_{+3}$ is the unrestricted weighted least squares weighted average that adjusts the degrees of freedom. It is generally less bias than meta-analyses that transform PCCs to Fisher's z.

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1. INTRODUCTION

Hundreds of meta-analyses of partial correlation coefficients (PCCs) are conducted each year widely across economics, business, education, psychology, and medical research.[i] Some researchers consider partial correlations to be the preferred effect size to summarize multiple regressions.[1] Others recommend using partial correlations as a last resort when different measures of the dependent variable and/or the independent variable of interest are routinely employed in the relevant area of research.[2]

It is widely known that individual correlation estimates, and PCCs, are biased downward (e.g., Olkin and Pratt).[3] Recently, Stanley and Doucouliagos uncover the counterintuitive result that all meta-analyses of PCC are, in contrast, biased upward.[4] That is, all meta-analyses of PCCs are biased regardless of whether fixed effect (FE), random effects (RE), or the unrestricted weighted least squares (UWLS) weighted average are employed and in the absence of any publication selection bias.[ii] In this paper, we offer novel small-sample corrections that render any remaining meta-analysis biases of PCCs scientifically trivial.

2. PARTIAL CORRELATION COEFFICIENTS

Across many disciplines, multiple regressions are employed to evaluate the effect of a treatment, condition, or variable upon some outcome of interest after controlling for other, potential contaminating, effects or obscuring complexities. Multiple regression can be represented as:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots + \beta_j X_{ji} + \varepsilon_i \qquad i = 1,2,\ldots,n, \tag{1}$$

[i] According to Google Scholar, 4,530 articles were published in 2022 that include the phrases "partial correlation" and "meta-analysis". Of course, not all of these studies are meta-analyses that use partial correlation coefficients. Some articles explain why they do not use partial correlations, while others are primary studies or narrative reviews citing meta-analyses. However, out of the first 100 hits, 75 are indeed meta-analyses that utilize partial correlations, as documented in our online appendix at meta-analysis.cz/pcc. It is probable that the proportion of meta-analyses using partial correlations among the Google Scholar hits will decrease further down the list. Nevertheless, even among the studies ranked between the 80th and 100th places, more than half are meta-analyses employing partial correlations. Hence, we have reason to believe that some hundreds of meta-analyses conducted in 2022 utilized partial correlations.

[ii] The unrestricted weighted least squares (UWLS) weighted average has been shown to have better statistical properties than RE when there is publication selection bias or when heterogeneity is correlated with sample size (or SE), which meta-research evidence finds in psychology.[14-16] Recently, UWLS is shown to better represent medical research than RE across over 67,000 meta-analyses of approximately 600,000 studies.[19]

where Y is the dependent variable or outcome of interest. Without loss of generalization, we take X_1 as the primary variable of interest (perhaps a dichotomous variable representing treatment). The other *X*s are independent variables that are thought to affect the outcome. Subscript *i* represents an induvial observation in a primary study (a consumer, an individual subject, a geographical region etc.), j is the total number of independent variables, and ε_i represent sampling errors and other residuals.

Multiple regression is used with observational data, quasi-experiments, and experimental designs when additional experimental conditions or pre-treatment subject characteristics need to be considered. For our purposes, the strength of the experimental design is not relevant as long as the focus of the meta-analysis is upon the estimated multiple regression coefficient, $\hat{\beta}_1$, across the research literature. However, in some cases, observational multiple regressions can offer strong research designs.[5]

The partial regression coefficient, $\hat{\beta}_1$, is not a standardized effect. It is measured in units of Y per a one unit increase in X_1. Any change in the measure, metric, or scale of either X_1 or Y from one study to the next will render the respective estimates of $\hat{\beta}_1$ uncomparable. Partial correlation coefficients solve this problem. PCCs have the same statistical properties and interpretation as simple bivariate correlations after the effects of $X_2, X_3, \dots, X_j$ have been eliminated.[6] Simple bivariate Pearson correlations are often employed as effect sizes in meta-analysis, and partial correlations come with the same advantages and limitations.

Gustafson mathematically derived a convenient formula that converts any partial regression coefficient, $\hat{\beta}_1$, into a partial correlation coefficient, r_p:

$$r_p = {t}\Big/{\sqrt{t^2 + df}}\,, \tag{2}$$

where $t = \frac{\hat{\beta}_1}{s_{\hat{\beta}_1}}$ is the conventional *t*-test for the statistical significance of X_1 in the explanation of Y, and $df = n - j - 1$ are the degrees of freedom available to the multiple regression, eq. (1).[7] r_p can be interpreted as a standardized regression coefficient that estimates the number of standard deviations that Y increases when X_1 increases by a one standard deviation, holding all other variables constant, and r_p^2 is the proportion of the variation in Y attributable to variation in X_1 after eliminating the effects of $X_2, X_3, \ldots, X_j$. Because economics, business, and social sciences, in general, often use different scales and measures for Y and/or X_1, PCCs are frequently employed in the meta-analysis of these fields.[2,8,9]

The variance of r_p is:

$$S_1^2 = {(1 - r_p^2)^2}\Big/{df} \qquad , \tag{3}$$

as derived in Olkin and Siotani.[1, 10, 11]

However, the test of PCC's statistical significance, H_0: $\rho = 0$, requires a slightly different formula for the variance of r_p:

$$S_2^2 = {(1 - r_p^2)}\Big/{df} \qquad ; \tag{4}$$

where ρ is the population partial correlation coefficient.[11,12] Otherwise, the test of statistical significance of the partial correlation would give an illogical and different result than the test of the statistical significance of the partial regression coefficient from which this PCC is derived.[2] Levy and Narula show that the more complex variance formula, S_1^2, reduces to S_2^2 when $\rho = 0$.[11,12] These two formulae for r_p's variance only differ in that the numerator of S_2^2 is not squared. Since, by definition, $-1 \leq r_p \leq 1$, it follows that $S_1^2 < S_2^2$ for all $|r_p| \neq \{0 \text{ or } 1\}$. Using S_2^2 and r_p reproduces the *t*-value and the *p*-value of the original estimated partial regression coefficient, $\hat{\beta}_1$; S_1^2 does not.

Below we demonstrate that all meta-analyses of PCCs are biased (including FE, RE, and UWLS) regardless of which formula of variance is used. Nevertheless, conventional meta-analyses that use S_1^2 cause the estimates of mean effect to be twice as biased as those which employ S_2^2. To address these biases, we offer a simple modification to the transformation formula, eq. (2), and a second small-sample bias correction for degrees of freedom. First, however, we establish and discuss the bias of the conventional meta-analysis of PCCs. It is only through understanding these biases that a solution can be found.

3. META-ANALYSIS BIAS

3.1 Simulations

To investigate the statistical properties of the meta-analysis of partial correlations, we conduct Monte Carlo simulations of RE and UWLS estimates of the mean PCC from randomly generated data, which is used to estimate multiple regressions and transform each $\hat{\beta}_1$ to a PCC. Simulations offer an important advantage over other approaches in that we can set the 'true' population value of the PCC, ρ, by forcing its value upon the data generating process.

To obtain estimated PCCs for the effect size corresponding to the variable, X_1, we start with the following multiple regression:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i \qquad i = 1,2,\ldots,n \tag{5}$$

n is set at {25, 50, 100, 200, or 400} but held constant for a given simulation to identify and understand the resulting small-sample biases. For simplicity, we set all betas to 1 and assume that X_{1i}, X_{2i}, and ε_i are independently and identically distributed as N(0,1).[iii] The variable, Y_i, is generated by eq. (5) after random and independent values are generated for X_{1i}, X_{2i}, and ε_i. As a next step, we estimate a multiple regression for eq. (5) and calculate the *t*-value of the estimated regression coefficient β_1. We then convert $X_1's$ *t*-value to a PCC via eq. (2).

Due to the clarity and simplicity of these data generating processes, the population variance of Y_i *not* attributed to the remaining independent variables, X_{2i}, equals 2 because this variance can be computed as the sum of the variances of X_{1i} and ε_i, each of which is set to have variance 1. Both X_{1i} and ε_i are *independently* distributed with variance 1; hence, this total variance is the sum of X_{1i} and ε_i variances. Thus, the ratio of $Y_i's$ remaining variance explained by X_{1i} is ½, leading to $\rho = \sqrt{½}$ or 0.707107. This result also follows from Gustafson where r_p^2 is shown to be: $\hat{\beta}_1^2 \Big/ \left(\hat{\beta}_1^2 + df \cdot S_{\hat{\beta}_1}^2\right)$.[5] Recall that β_1 is set to 1, $S_{\hat{\beta}_1}^2 = (\sigma^2 / df \cdot \sigma_{X_1}^2)$,[13] and both σ^2 and $\sigma_{X_1}^2$ are set to 1 by design; thus, again ρ^2 = ½. In other simulation experiments, we set ρ equal to a

[iii] We also simulate more complex multiple regression with 4, 6, and 10 independent variables. Results from these more complex multiple regressions are practically equivalent and are reported below and in the Supplement.

'medium' effect size (ρ = sqrt(.1) = .3162) by dividing X'_{1i} s randomly generated N(0,1) by 3 and a 'small' effect size (ρ = sqrt(1/82) = .1104) by dividing by 9. Doing so makes X'_{1i}s variance equal to 1/9 and 1/81, respectively while leaving the error variance at 1—see Table 1.

For each study in our simulations, all the data in eq. (5) is randomly generated, the multiple regression, eq. (5), and its coefficients are estimated, and r_p is calculated from eq. (2). S_1^2 is then calculated from eq. (3) and S_2^2 from eq. (4), and all these calculations are repeated 50 times to represent one meta-analysis.[iv] For each meta-analysis of 50 estimated PCCs, the RE and the UWLS weighted averages are calculated in two ways by using S_1^2 and S_2^2.

UWLS estimates the simple meta-regression coefficient, α_1, from:

$$t_k = \frac{r_{pk}}{SE_k} = \alpha_1\left(\frac{1}{SE_k}\right) + u_k \qquad k = 1, 2, \ldots, 50 \qquad (6)$$

k is the number of PCCs combined into the meta-analysis; k is often called the number of studies. In the supplement, we also report the results for the simulation designs that correspond to Tables 2 and 4 but with k={10; 200} to ensure robustness. SE_k is calculated as the square root of either S_1^2 or S_2^2 from their respective formulae above. Any common statistical software automatically calculates UWLS, $\hat{\alpha}_1$, its standard error, test statistic, and confidence intervals. UWLS and the fixed effect (FE) must have identical point estimates, but UWLS automatically adjusts its standard errors and confidence intervals for heterogeneity when present.[14,15] We do not assume a common effect but instead allow for random, additive heterogeneity (Section 3.3, below); thus, FE is not an appropriate model for these simulations. Previous simulations have shown that UWLS is statistically superior to RE if there is selection for statistical significance or if small studies are more heterogeneous than larger studies.[14,16] In other cases where RE's model is imposed upon the simulations, the differences between UWLS' and RE's statistical properties are negligible. For each randomly generated meta-analysis, the bias, RMSE (square root of the mean squared error)

[iv] These biases are largely independent of the number of PCCs (k) in the meta-analysis, but very dependent on the sample size (n) of the primary study. Stanley and Doucouliagos used other values of k and found that meta-analyses of 10 or fewer studies consistently have slightly smaller biases while those with a larger number of estimates (k = 200) have slightly larger biases. Thus, the pattern and size of these small-sample biases are largely independent of the number of PCCs (k) in the meta-analysis.[4]

and coverage rates of RE and UWLS are calculated and then averaged across 10,000 replications of all these steps. See the Supplement for the simulation code.

Table 1 reports the results of these simulations using both versions of PCC's variance—eq. (3) and eq. (4). Using either RE or UWLS with S_2^2 consistently produces biases only 50% as large as the conventional approach, RE with S_1^2, on average and for most of the individual conditions. Table 1 also shows that S_1^2 generates larger root mean squared errors and worse coverage (*i.e.*, coverage rates that are often much different than their nominal 95% level) than S_2^2. In Section 3.2, below, we discuss the reason for these biases and why S_1^2 produces predictably larger biases. These results confirm Stanley and Doucouliagos' finding that the theoretically 'correct' variance, S_1^2, eq. (3), is not useful in practice when conducting meta-analyses of partial correlations.[4]

3.2 Reducing meta-analysis bias to triviality

Looking closely at the biases identified through simulations reveals two additional lessons. First, although these biases are of a notable magnitude for small samples ($n \leq 50$), all these biases are mere rounding errors (*i.e.*, < .005) or smaller for large samples (*i.e.*, $n \geq 200$ or $n \geq 100$ if S_2^2 is used). Second, biases consistently halve as n doubles. Figure 1 graphs RE's and UWLS' biases against the inverse of degrees of freedom ($1/df$) when $\rho = \sqrt{½}$, using 10,000 replications of each sample size, n = {10, 20, 40, 80, 160, 320, 640, 1280 & 25, 50, 100, 200, 400, 800, 1600, 2500}. Figure 1 reveals that S_2^2 approximately halves RE's bias and that doubling the sample size of the original study halves the bias of each again.

To be more precise, the biases of UWLS with inverse S_2^2 weights are a near exact function of the inverse of degrees of freedom ($1/df$):

$$Bias_i = .000069 + .508\left(\frac{1}{df_i}\right) \tag{7}$$

$$t = (1.67) \quad (505.8) \qquad ; \ R^2 = .9999453$$

The values in parentheses are the t-values for the estimated regression intercept and slope coefficients, respectively; values greater than 2.145 are statistically significant at the .05 level (t-values with df=14). The inverse of degrees of freedom, $\left(\frac{1}{df_i}\right)$, explains over 99.99% of the bias of

UWLS ($R^2 \approx 99.995\%$) leaving a 95% margin of error of .0003. Through numerical analysis, we know that the bias of the meta-analysis of PCCs is a function of the inverse *df*, and that any remaining error is negligible.

A century ago, Fisher observed that the: "sampling distribution of the partial correlation obtained from *n* pairs of values, when one variable is eliminated, is the same as the random sampling distribution of a total correlation derived from (*n*-1) pairs. By mere repetition of the above reasoning, it appears that when *s* variates are eliminated the effective size of the sample is diminished to (*n*-*s*)" (p. 330).[6] This suggests that fine-tuning the degrees of freedom in PCC's transformation formula may substantially reduce or practically eliminate this bias. Further simulations confirm that this is indeed the case.

3.2.1 Reducing meta-analysis of PCCs bias to triviality: RE_{ss}

Following Fisher's observation, consider the simple bivariate correlation:

$$r = \frac{S_{xy}}{S_x \cdot S_y} = \sum(X_i - \bar{X})(Y_i - \bar{Y}) \Big/ \sqrt{\sum(X_i - \bar{X})^2} \cdot \sqrt{\sum(Y_i - \bar{Y})^2} \quad . \qquad (8)$$

The sample covariance, S_{xy}, has degrees of freedom (n-2), because two parameters, μ_x and μ_y, must be first estimated from a sample of n pairs of observations. Each sample variance, S_x^2 and S_y^2, has (n-1) has degrees of freedom; thus, the denominator is (n-1). This suggests that a correction for degrees of freedom, (n-2)/(n-1), might reduce the small-sample bias of meta-analysis weighted averages that is revealed in Table 1. When the small-sample bias is proportional to 1/*df* and *df* = (n-1) multiplying by (n-2)/(n-1) should reduce or correct this small-sample bias. Table 2 reports the random-effects, small-sample correction, RE_{ss}, where each sample PCC is first multiplied by (n-2)/(n-1) before the usual random-effects formulae are applied. RE_{ss} greatly reduces the small-sample biases—compare Tables 1 and 2.

These small-sample corrections of PCCs, however, *should not* be applied to individual stand-alone PCCs because it is widely known that individual correlation estimates, and PCCs, are biased downward.[3] Applying this small-sample adjustment to stand-alone PCCs would then only make a small downward bias worse. Rather, they should be used only as an intermediate step in the calculations of meta-analysis weighted averages of PCCs. We propose employing these small-sample corrections, (n-2)/(n-1) and $UWLS_{+3}$ (see below) only in the calculations of meta-analysis weighted averages of PCCs. This small sample correction could be applied to UWLS as well, but we have found a better and more direct way to adjust degrees of freedom for UWLS—$UWLS_{+3}$.

3.2.2 Reducing meta-analysis of PCCs bias to triviality: $UWLS_{+3}$

As shown above, eq. (7), the biases of these meta-analysis estimators are nearly an exact function of the inverse degrees of freedom. Note further that df is in the denominator of Gustafson's PCC transformation formula, eq. (2), making all PCCs an inverse function of the degrees of freedom. This suggests that a simple adjustment of df in eq. (2) might provide a solution. Numerical analysis finds that adding 3 to df successfully reduces these small-sample biases to scientific triviality. We call the resulting transformed weighted average '$UWLS_{+3}$.'

$UWLS_{+3}$ substitutes degrees of freedom that are three larger than the multiple regression's degrees of freedom into PCC's transformation formula, eq. (2). That is, $UWLS_{+3}$ calculates PCCs as:

$$r_p = t \Big/ \sqrt{t^2 + df_{+3}} \qquad (9)$$

for $df_{+3} = n - s + 1$ with *s* as the number of independent variables in the multiple regression held constant in the calculation of the partial correlation of interest (*i.e.*, $s = j - 1$). We do not recommend that this transformation be used in conjunction with random effects, as this produces worse statistical properties in some conditions.

3.2.3 Simulation findings

UWLS+3 employs the same simulation design as before; however, it replaces the degrees of freedom in the partial correlation coefficient's transformation formula with values that are three units greater than the degrees of freedom in the multiple regression. As displayed in Table 2, $UWLS_{+3}$ eliminates all biases to within < $\underline{+}$.001, and its average absolute bias is only .0002. RE_{ss} also greatly reduces these biases, but not to the extent that $UWLS_{+3}$ does, nor are RE_{ss} coverages as close to 95% as are $UWLS_{+3}$'s. Table 2 assumes that either there are two independent variables in the multiple regression ($j = 2$) or four ($j = 4$). To ensure broader generalizability, Supplement Table S1 reports the same simulation design as Table 2, except j = 6 &10. Induction suggests that if you can prove trivial bias for one (*i.e., s* =1; Table 2) and trivial bias for some random *s* (*e.g.,* s = 3), then trivial biases generalize to any *s* (*e.g., s* = {5, 9}, Table S1). As a further corroboration of the effective elimination of meta-analysis bias, Table S2 reports the same simulation design but with different values of the population partial correlation coefficient, ρ ={.9487; .2425; 0}. Also note Table S3 where the same simulation design is reported but with different numbers of PCCs, k={10; 200}. In all cases, these adjustments drive the small-sample biases to scientific negligibility and their relative evaluations remain unchanged.

Now that we have found ways to reduce these biases to scientific triviality, what causes these biases of the conventional meta-analysis of partial correlations? The simple answer is that both formulas for the variance of PCCs are themselves a function of the PCC. Because the weights of meta-analysis are a strictly increasing function of r_p^2, it follows that for all $r_p^2 \neq$ {0 or 1} positive sampling errors are assigned more influence on the meta-analysis estimate compared to negative sampling errors of the same magnitude. In all meta-analyses that use inverse variance weights, based on either S_1^2 or S_2^2, an upwards bias in magnitude will arise: the absolute expected value delivered by the meta-analysis will surpass | ρ | if the true correlation is not 0 or 1.

Let us assume, for instance, that ρ = 0.7 and examine how estimates with errors of the same magnitude but different signs ($\pm$ 0.2) are weighted in meta-analysis. For S_2^2, an UWLS estimate with a sampling error of +0.2 is assigned a weight proportional to 1/.19 = 5.26, in stark contrast to 1/.75 = 1.333 for a -0.2 sampling error. Here estimates with positive errors are assigned nearly 4 times more influence than estimates with negative errors but equal in size. Few sampling errors will in practice be as large as $\pm$ 0.2, but the aforementioned principle of asymmetric weighting as the source of bias in conventional meta-analysis of partial correlations holds in general: for all sizes of sampling errors and various meta-analysis estimators. Because RE's weights are the inverse of the sampling variance plus a positive constant (τ^2), this asymmetric weighting of sampling errors is moderated, but not eliminated, by RE. Table 1 shows that RE's biases are somewhat smaller than UWLS', just as we would expect, and these differences are especially clear for small samples when S_1^2 is used. Asymmetric weighting of sampling errors biases weighted averages upwards in magnitude. Table 1 confirms these biases.

For bivariate correlations, this issue that the variance is a function of the effect size and that this may be problematic for meta-analysis is widely known. The conventional solution is to convert correlations to Fisher z's, calculate the meta-analysis estimate of the mean and its related statistics, then convert these terms of Fisher z back to correlations for the purpose of interpretation.[17] As Fisher noted, what is true for correlations is true for partial correlations after degrees of freedom are adjusted for the number of variables eliminated, *s*.[6] Tables 2, S1-S3 also report the biases, RMSEs, and coverage rates for random effect estimates of Fisher's z that have been converted back to PCCs. Using Fisher's z eliminates most of these small-sample biases. Its biases and MSEs are nearly the same as the simple RE correction for small-sample bias. However, in all cases and by all criteria, $UWLS_{+3}$, has better statistical properties than either Fisher's z or RE_{ss} (Table 2). Although Fisher's z and RE_{ss} produce biases larger than rounding error only for small samples and medium or larger correlations, $UWLS_{+3}$'s bias is still ten times smaller, see Figure 2. Likewise, $UWLS_{+3}$'s RMSEs are smaller, and its coverage rates are closer to the nominal 95% than Fisher's z or RE_{ss}. In fact, RE_{ss} CIs are too narrow for large PCCs. Practically speaking, however, all three: Fisher's z, RE_{ss}, and $UWLS_{+3}$ solve this problem of biased meta-analyses of partial correlations in the vast majority of cases even though $UWLS_{+3}$ is slightly better.

3.3 Heterogeneity

Notable heterogeneity across studies within an area of research is common in all disciplines. In psychology, for example, the observed variance from study-to-study is about 4 times larger than what reported standard errors imply (*i.e.*, median I^2 = 74%).[18] To ensure that partial correlation's biases are robust to heterogeneity, we have modified the same simulation design to produce heterogeneity at levels seen in psychology. Tables 3 and 4 report the same simulations as Tables 1 and 2, except that random heterogeneity is added to each study's estimated correlation in each meta-analysis. We first convert each randomly generated estimated PCC to Cohen's d, add a random normal deviation with mean zero and standard deviation {.5, .3, .2*d*} as ρ is: {0.7071, 0. 3162, 0.1104}, and, lastly, transform this back to a partial correlation. That is, the simulations fix tau to be {.5, .3, .2*d*} as ρ is: {0.7071, 0. 3162, 0.1104}. We transform to Cohen's d in this way to produce random heterogeneity consistent with the random-effect model and to reproduce roughly the same distribution of heterogeneity as seen in psychology, in both absolute terms (d) and relatively (I^2).[v] Table 3 shows that the biases of the meta-analysis of correlations remain, while Table 4 confirms that Fisher's z and the small-sample corrections introduced here consistently reduce these biases to scientific negligibility.

4. DISCUSSION

Meta-analyses of partial correlation coefficients (PCC) are generally biased. We offer new solutions: $UWLS_{+3}$ and the small-sample correction, RE_{ss}. Although these biases are ubiquitous, the good news is that they practically and scientifically disappear when the primary studies employ larger samples ($n \geq 200$). Thus, these biases will typically not be a notable factor in the meta-analysis of econometric studies in economics and finance, which often involve hundreds of observations or more.[vi] Nonetheless, for many areas of education, business, psychology, medicine and health, meta-analysts should use $UWLS_{+3}$, RE_{ss}, or Fisher's z in the meta-analysis of PCCs.

An important limitation to our study is that the primary research literatures will typically be much richer than what our simulations have assumed. We abstract from such complexities to isolate and detect these biases and then to understand their underlying cause. However, many meta-analyses will include some studies which may be sufficiently large to have negligible bias, which will likely moderate the biases of these weighted averages. Thus, in most social science applications, it is unlikely that the bias of the meta-analysis of partial correlation coefficients will be as large as those revealed here in small samples.

Both $UWLS_{+3}$ and RE_{ss} are easy to implement. To calculate $UWLS_{+3}$, meta-analysts merely need to add 3 to *df* in PCC's transformation formula, eq. (2), and use the formula that calculates PCC's variance, S_2^2, eq. (4). $UWLS_{+3}$ is the simple regression coefficient, eq. (6), and it can be estimated using any regression software. Note that UWLS' regression does not have an intercept

[v] Generating heterogeneity though random variations to X_1's regression coefficient, β_1=1 $\pm$ N(0, .2) produces approximately same overall results as Table 3 and Table 4.
[vi] Across 358 economic meta-analyses about 2/3[rds] of 174,542 estimates are computed from sample sizes larger than 200.[20]

(or a ‘constant’). Aside from small improvements to bias, MSE, and coverage rates over Fisher’s z,[vii] UWLS_{+3}’s advantage lies in its computational simplicity and the clarity of its interpretation.

Unlike the meta-analysis of Fisher’s z, UWLS_{+3} is a partial correlation and can be understood entirely as such. Neither UWLS_{+3} nor RE_{ss} need to be transformed back to a correlation to be interpretable. This is particularly helpful for multiple meta-regression analysis (MRA). In economics applications, meta-analyses of PCCs are common and frequently involve a dozen or more moderator variables. To understand the impact of important MRA coefficients, it is necessary to interpret them in terms of the effect size studied, in this case partial correlation coefficients. When Fisher’s zs are the object of meta-analysis and MRA, it is easy to misinterpret MRA results as correlations. With multiple MRA, the inverse Fisher’s z transformation, $\mathrm{PCC} = e^{\left[\frac{2\cdot Z-1}{2\cdot Z+1}\right]}$, would need to be separately employed multiple times if Fisher’s zs are meta-analyzed.

Computational simplicity and clarity of interpretation are also advantages of RE_{ss}. When there is little or no heterogeneity, Table 2, UWLS_{+3} dominates both Fisher’s z and RE_{ss}. However, RE_{ss} has a limitation not seen in either UWLS_{+3} or Fisher’s z. When the ‘true’ correlation is very large, ρ = .9487, RE_{ss} has notably larger biases than either UWLS_{+3} or Fisher’s z. However, we have not seen average PCCs as large .7 in any economics meta-analysis,[viii] and no bivariate average correlation (RE) has an absolute value larger than .6 among the 108 *Psychological Bulletin* meta-analyses.[18]

V. CONCLUSION

We find that all meta-analyses of partial correlations are biased, and we offer simple remedies for these biases, UWLS_{+3} and RE_{ss}. Both make a simple adjustment to the degrees of freedom used to calculate partial correlations and thereby render trivial any remaining bias. UWLS_{+3} generally outperforms RE_{ss} and the more cumbersome application of Fisher’s z, but all three reduce bias to trivial magnitudes in the great majority of practical applications. Our simulations also reveal that all biases are small-sample biases ($n \leq 200$). Thus, in applications where primary studies typically

[vii] When there is heterogeneity and a relatively large number of studies (k=200), Fisher’s z and UWLS_{+3} have virtually the same statistical properties—see Table S4.
[viii] Among 151 meta-analyses of partial correlations for which we have data, the UWLS estimate ranges from -0.45 to 0.55. The median absolute UWLS is 0.021.[20]

have hundreds and even more observations, PCCs can be meta-analyzed in any of the above ways without notable bias. However, for many fields in the social and the medical sciences where small-sample studies dominate, these small-sample biases are easily avoidable by employing $UWLS_{+3}$, RE_{ss}, or Fisher's z.

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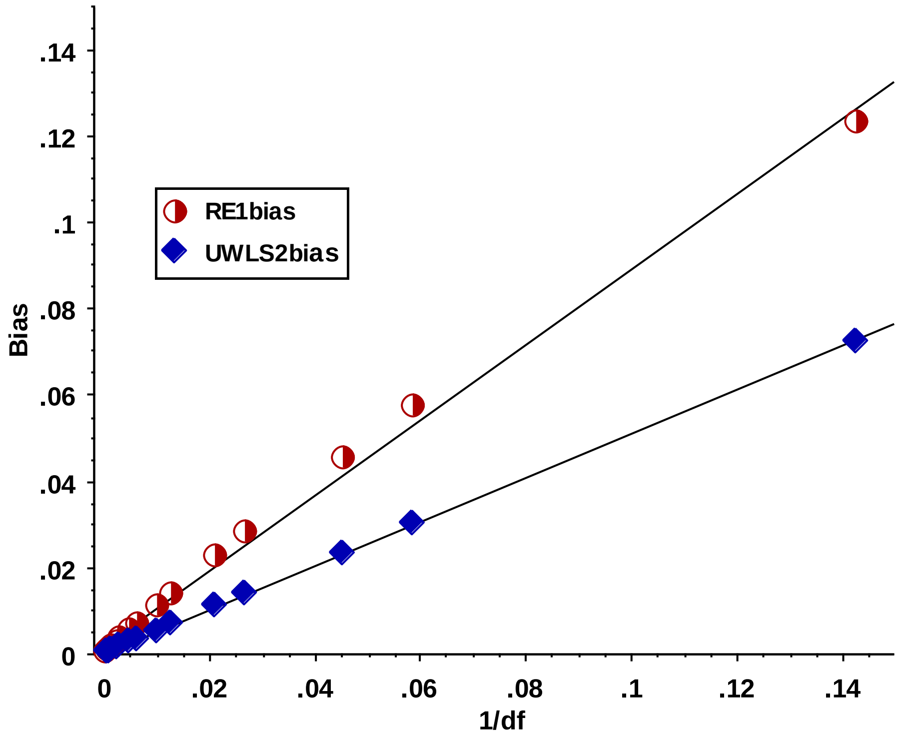

FIGURE 1: Biases of random-effects and the unrestricted weight least square. Each point represents an average bias across 10,000 replications. **RE1bias** is random effects' bias that use PCC variance, S_1^2, from eq. (3). **UWLS2bias** is UWLS' bias using S_2^2 from eq. (4).

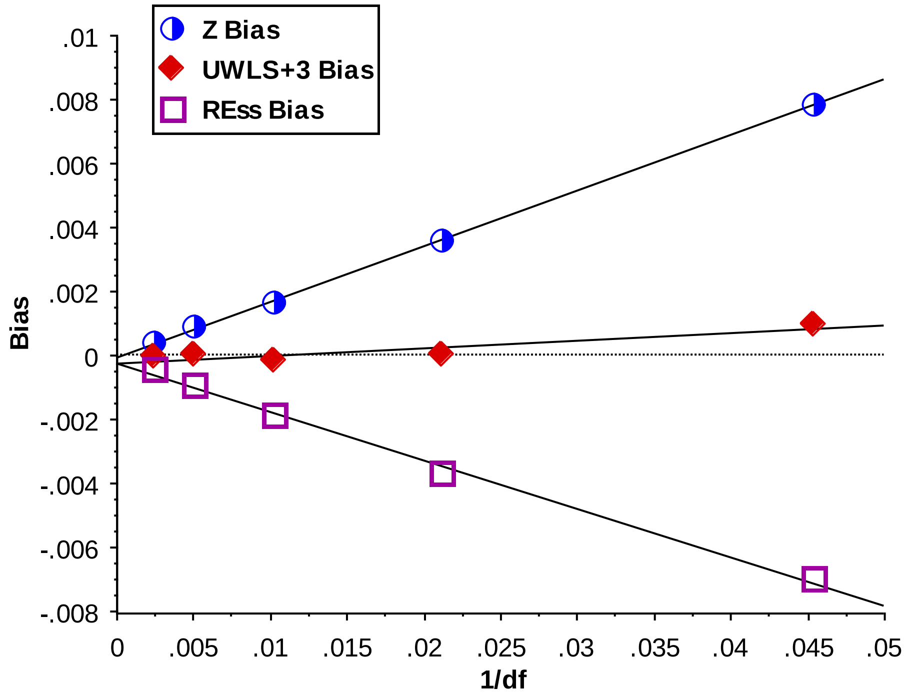

FIGURE 2: Biases of the meta-analysis of Fisher's z converted back to PCC (**Z Bias**), the unrestricted weight least squares with 3 additional degrees of freedom (**UWLS+3**), and the random-effect's estimate of the mean, **RE**$_{ss}$, using S_2^2, from eq. (3) and the small-sample adjustment (n-2)/(n-1) for $\rho = \sqrt{½}$ and 10,000 replications. See Table 2 and its discussion.

Table 1: The meta-analyses of PCCs (RE and UWLS) using different formulae for PCC's variance

Design		**Bias**				**RMSE**				**Coverage**			
ρ	**n**	**RE$_1$**	**RE$_2$**	**UWLS$_1$**	**UWLS$_2$**	**RE$_1$**	**RE$_2$**	**UWLS$_1$**	**UWLS$_2$**	**RE$_1$**	**RE$_2$**	**UWLS$_1$**	**UWLS$_2$**
.7071	25	.0455	.0233	.0540	.0233	.0478	.0278	.0568	.0278	.1428	.8521	.0588	.3787
.7071	50	.0223	.0108	.0254	.0108	.0245	.0149	.0277	.0149	.4103	.9497	.2954	.5928
.7071	100	.0111	.0053	.0125	.0053	.0131	.0088	.0145	.0088	.6619	.9796	.5788	.7136
.7071	200	.0055	.0026	.0061	.0026	.0075	.0057	.0080	.0057	.8109	.9878	.7714	.7734
.7071	400	.0028	.0013	.0031	.0013	.0045	.0038	.0048	.0038	.8824	.9911	.8585	.8025
.3162	25	.0347	.0173	.0490	.0194	.0461	.0336	.0591	.0348	.7358	.8987	.5843	.8312
.3162	50	.0179	.0083	.0216	.0089	.0265	.0208	.0295	.0211	.8327	.9329	.7810	.8900
.3162	100	.0091	.0042	.0104	.0045	.0161	.0138	.0170	.0139	.8892	.9469	.8714	.9118
.3162	200	.0045	.0020	.0050	.0022	.0102	.0093	.0105	.0093	.9246	.9612	.9127	.9278
.3162	400	.0022	.0009	.0024	.0010	.0068	.0065	.0069	.0065	.9424	.9599	.9339	.9349
.1104	25	.0134	.0065	.0198	.0079	.0360	.0321	.0412	.0328	.9114	.9413	.8771	.9234
.1104	50	.0073	.0034	.0088	.0039	.0225	.0208	.0234	.0210	.9332	.9517	.9246	.9410
.1104	100	.0034	.0015	.0040	.0017	.0150	.0144	.0152	.0145	.9431	.9532	.9362	.9430
.1104	200	.0017	.0007	.0019	.0008	.0102	.0100	.0103	.0100	.9495	.9548	.9424	.9468
.1104	400	.0009	.0005	.0010	.0005	.0071	.0070	.0071	.0070	.9596	.9623	.9533	.9535
Average		.0122	.0059	.0150	.0063	.0196	.0153	.0221	.0155	.7953	.9482	.7520	.8310

Notes: ρ is the 'true' population mean partial correlation coefficient (PCC). **n** is the sample size used in the primary study's multiple regression. **Bias** is the difference between the meta-analysis estimate and ρ calculated from 50 estimated partial correlation coefficients and averaged across 10,000 replications. **RMSE** is the square root of the mean squared error. ***Coverage*** is the proportion of 10,000 meta-analyses' 95% confidence intervals that contain ρ. **RE** is the random-effect's estimate of the mean, and **UWLS** is the unrestricted weighted least squares' estimate of the mean. The subscripts (1 and 2) refer to the use of either the PCC variance, S_1^2, from eq. (3) or S_2^2 from eq. (4) to calculate the RE and UWLS weighted averages.

Table 2: **RE**$_{ss}$, **RE**$_{z}$, and **UWLS**$_{+3}$ meta-analyses of partial correlations

2 IVs: Partial Correlation of X_1 from $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i$										
Design		**Bias**			**RMSE**			**Coverage**		
ρ	**n**	**RE**$_{ss}$	**RE**$_{z}$	**UWLS**$_{+3}$	**RE**$_{ss}$	**RE**$_{z}$	**UWLS**$_{+3}$	**RE**$_{ss}$	**RE**$_{z}$	**UWLS**$_{+3}$
.7071	25	-.0070	.0078	.0009	.0161	.0168	.0155	.9891	.9281	.9431
.7071	50	-.0037	.0036	.0001	.0107	.0109	.0105	.9914	.9460	.9511
.7071	100	-.0019	.0017	-.0001	.0075	.0073	.0072	.9923	.9530	.9514
.7071	200	-.0010	.0008	-.0001	.0051	.0051	.0051	.9938	.9539	.9503
.7071	400	-.0004	.0004	.0000	.0035	.0036	.0036	.9953	.9551	.9480
.3162	25	.0050	.0067	.0008	.0281	.0284	.0275	.9516	.9492	.9408
.3162	50	.0017	.0032	.0003	.0188	.0190	.0187	.9569	.9519	.9458
.3162	100	.0008	.0014	.0000	.0129	.0131	.0130	.9626	.9553	.9460
.3162	200	.0005	.0006	-.0002	.0091	.0091	.0091	.9646	.9567	.9482
.3162	400	.0002	.0004	.0000	.0063	.0064	.0064	.9659	.9556	.9497
.1104	25	.0016	.0024	.0002	.0306	.0306	.0301	.9478	.9545	.9368
.1104	50	.0007	.0011	.0000	.0208	.0206	.0203	.9496	.9593	.9481
.1104	100	.0004	.0007	.0001	.0143	.0143	.0142	.9527	.9584	.9489
.1104	200	.0003	.0002	-.0001	.0099	.0100	.0100	.9573	.9569	.9485
.1104	400	.0001	.0001	-.0001	.0069	.0071	.0070	.9609	.9564	.9495
Average		.0017[a]	.0021	.0002[a]	.0134	.0135	.0132	.9688	.9527	.9471
4 IVs: Partial Correlation of X_1 from $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \varepsilon_i$										
.7071	25	-.0048	.0083	.0009	.0160	.0163	.0164	.9920	.9284	.9424
.7071	50	-.0032	.0037	-.0001	.0108	.0107	.0106	.9930	.9434	.9447
.7071	100	-.0017	.0018	-.0001	.0074	.0073	.0073	.9929	.9513	.9512
.7071	200	-.0009	.0008	-.0001	.0051	.0050	.0050	.9949	.9554	.9506
.7071	400	-.0004	.0004	.0000	.0036	.0036	.0036	.9935	.9556	.9490
.3162	25	.0064	.0063	.0000	.0297	.0289	.0289	.9491	.9520	.9380
.3162	50	.0020	.0029	-.0001	.0192	.0191	.0191	.9551	.9545	.9456
.3162	100	.0008	.0014	-.0001	.0131	.0129	.0130	.9606	.9588	.9516
.3162	200	.0005	.0006	-.0001	.0090	.0091	.0092	.9658	.9592	.9518
.3162	400	.0002	.0003	-.0001	.0064	.0063	.0065	.9642	.9591	.9554
.1104	25	.0025	.0029	.0005	.0325	.0312	.0316	.9440	.9553	.9379
.1104	50	.0010	.0012	.0000	.0212	.0209	.0209	.9508	.9580	.9463
.1104	100	.0004	.0007	.0001	.0145	.0144	.0145	.9548	.9553	.9473
.1104	200	.0001	.0002	-.0001	.0102	.0100	.0101	.9508	.9562	.9472
.1104	400	-.0001	.0001	.0000	.0070	.0071	.0071	.9597	.9543	.9458
Average		.0017[a]	.0021	.0002[a]	.0137	.0138	.0135	.9681	.9531	.9470

Notes: ρ is the 'true' population mean partial correlation coefficient (PCC). **n** is the sample size used in the primary study's multiple regression. **Bias** is the difference between the meta-analysis estimate and ρ calculated from 50 estimated partial correlation coefficients and averaged across 10,000 replications. **RMSE** is the square root of the mean squared error. ***Coverage*** is the proportion of 10,000 meta-analysis 95% confidence intervals that contain ρ. **RE**$_{ss}$ is the random-effect's estimate of the mean using S_2^2, from eq. (3) and the small-sample adjustment (n-2)/(n-1). **UWLS**$_{+3}$ is the unrestricted weighted least squares' estimate of the mean using S_2^2 from eq. (4) and df_{+3} as the degrees of freedom in PCC's formula. **RE**$_{z}$ is the random-effect's estimate of Fisher's z converted back to PCC. [a]Average biases are averages across the absolute values of the biases. Biases reported as '.0000' are < |±.00005|.

Table 3: The meta-analyses of PCCs (RE and UWLS) using different formulae for PCC's variance and with heterogeneity

Design		Bias				RMSE				Coverage			
ρ	I^2	**RE_1**	**RE_2**	**$UWLS_1$**	**$UWLS_2$**	**RE_1**	**RE_2**	**$UWLS_1$**	**$UWLS_2$**	**RE_1**	**RE_2**	**$UWLS_1$**	**$UWLS_2$**
.7071	.369	.0385	.0245	.0710	.0270	.0435	.0317	.0736	.0328	.3931	.7546	.0322	.4151
.7071	.559	.0124	.0068	.0459	.0149	.0214	.0198	.0485	.0216	.7771	.8724	.1362	.6138
.7071	.731	-.0012	-.0045	.0347	.0095	.0156	.0168	.0374	.0169	.9018	.9143	.2611	.7180
.7071	.848	-.0086	-.0105	.0292	.0069	.0171	.0184	.0320	.0149	.8657	.8746	.3571	.7586
.7071	.920	-.0125	-.0136	.0268	.0058	.0190	.0198	.0296	.0140	.7970	.8217	.4035	.7753
.3162	.404	.0241	.0105	.0601	.0209	.0429	.0355	.0715	.0396	.8424	.9134	.5489	.8360
.3162	.516	.0087	.0011	.0343	.0109	.0285	.0266	.0445	.0287	.9099	.9354	.7167	.8845
.3162	.668	.0004	-.0036	.0232	.0064	.0225	.0225	.0330	.0233	.9396	.9396	.8015	.9116
.3162	.801	-.0038	-.0058	.0184	.0045	.0205	.0209	.0279	.0207	.9459	.9404	.8370	.9224
.3162	.890	-.0061	-.0071	.0159	.0034	.0202	.0205	.0257	.0198	.9312	.9282	.8543	.9203
.1104	.319	.0108	.0049	.0217	.0079	.0378	.0346	.0457	.0360	.9182	.9334	.8641	.9168
.1104	.363	.0049	.0015	.0108	.0037	.0263	.0251	.0293	.0257	.9332	.9398	.9102	.9343
.1104	.498	.0017	-.0001	.0063	.0019	.0204	.0200	.0221	.0204	.9336	.9352	.9242	.9342
.1104	.661	.0001	-.0008	.0044	.0012	.0170	.0169	.0182	.0172	.9447	.9448	.9344	.9415
.1104	.795	-.0010	-.0015	.0032	.0006	.0156	.0156	.0165	.0158	.9435	.9410	.9369	.9419
Average		.0090[a]	.0065[a]	.0271	.0084	.0245	.0230	.0370	.0232	.8651	.9059	.6346	.8283

Notes: ρ is the 'true' population mean partial correlation coefficient (PCC). Sample sizes as the same as reported in Tables 1 and 2. $0 \leq \mathbf{I^2} \leq 1$ is a relative measure of heterogeneity. **Bias** is the difference between the meta-analysis estimate and ρ calculated from 50 estimated partial correlation coefficients and averaged across 10,000 replications. **RMSE** is the square root of the mean squared error. ***Coverage*** is the proportion of 10,000 meta-analyses' 95% confidence intervals that contain ρ. **RE** is the random-effect's estimate of the mean, and **UWLS** is the unrestricted weighted least squares' estimate of the mean. The subscripts (1 and 2) refer to the use of either the PCC variance, S_1^2, from eq. (3) or S_2^2 from eq. (4) to calculate the RE and UWLS weighted averages. [a]Average biases are averages across the absolute values of the biases.

Table 4: **RE$_{ss}$**, **RE$_z$**, and **UWLS$_{+3}$** meta-analyses of partial correlations with heterogeneity

2 IVs: Partial Correlation of X_1 from $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i$										
Design		**Bias**			**RMSE**			**Coverage**		
ρ	**I^2**	**RE$_{ss}$**	**RE$_z$**	**UWLS$_{+3}$**	**RE$_{ss}$**	**RE$_z$**	**UWLS$_{+3}$**	**RE$_{ss}$**	**RE$_z$**	**UWLS$_{+3}$**
.7071	.369	-.0058	.0024	.0041	.0199	.0199	.0203	.9614	.9404	.9465
.7071	.559	-.0068	-.0016	.0043	.0192	.0165	.0167	.9110	.9429	.9378
.7071	.730	-.0113	-.0038	.0043	.0198	.0152	.0149	.8717	.9392	.9397
.7071	.848	-.0140	-.0046	.0045	.0205	.0145	.0140	.8233	.9340	.9333
.7071	.919	-.0154	-.0053	.0044	.0210	.0144	.0136	.7897	.9279	.9317
.3162	.404	-.0004	.0037	.0020	.0333	.0327	.0331	.9305	.9421	.9388
.3162	.515	-.0049	.0001	.0018	.0265	.0256	.0261	.9328	.9470	.9456
.3162	.669	-.0068	-.0013	.0022	.0233	.0222	.0226	.9316	.9427	.9447
.3162	.800	-.0075	-.0022	.0022	.0215	.0204	.0207	.9274	.9398	.9416
.3162	.890	-.0077	-.0025	.0023	.0204	.0190	.0192	.9270	.9430	.9461
.1104	.320	.0012	.0018	.0003	.0326	.0334	.0335	.9413	.9461	.9373
.1104	.364	-.0006	.0005	.0003	.0245	.0248	.0249	.9405	.9427	.9417
.1104	.500	-.0006	.0001	.0004	.0193	.0199	.0201	.9460	.9415	.9440
.1104	.661	-.0010	-.0001	.0006	.0167	.0170	.0172	.9449	.9445	.9482
.1104	.795	-.0014	-.0004	.0004	.0154	.0154	.0155	.9450	.9460	.9506
Average		.0057[a]	.0020[a]	.0023	.0223	.0207	.0208	.9149	.9413	.9418
4 IVs: Partial Correlation of X_1 from $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \varepsilon_i$										
.7071	.349	-.0031	.0033	.0044	.0195	.0206	.0209	.9671	.9372	.9422
.7071	.549	-.0062	-.0016	.0042	.0191	.0165	.0167	.9183	.9459	.9430
.7071	.726	-.0110	-.0039	.0042	.0195	.0152	.0148	.8738	.9402	.9421
.7071	.847	-.0139	-.0049	.0043	.0203	.0147	.0140	.8284	.9331	.9367
.7071	.919	-.0152	-.0050	.0048	.0208	.0141	.0135	.7963	.9325	.9326
.3162	.398	.0008	.0048	.0025	.0347	.0338	.0342	.9272	.9461	.9386
.3162	.508	-.0041	.0005	.0021	.0267	.0259	.0264	.9348	.9440	.9433
.3162	.665	-.0069	-.0016	.0018	.0232	.0222	.0225	.9311	.9425	.9439
.3162	.800	-.0073	-.0019	.0025	.0213	.0202	.0205	.9323	.9454	.9465
.3162	.889	-.0081	-.0023	.0026	.0207	.0192	.0195	.9262	.9413	.9433
.1104	.323	.0012	.0020	.0004	.0344	.0346	.0346	.9392	.9473	.9365
.1104	.358	-.0001	.0007	.0004	.0247	.0251	.0252	.9410	.9437	.9421
.1104	.495	-.0010	.0005	.0009	.0199	.0198	.0200	.9392	.9446	.9462
.1104	.658	-.0011	-.0005	.0002	.0167	.0171	.0173	.9403	.9390	.9431
.1104	.794	-.0014	-.0004	.0005	.0153	.0154	.0156	.9451	.9410	.9457
Average		.0054[a]	.0023[a]	.0024	.0224	.0209	.0210	.9160	.9416	.9417

Notes: ρ is the 'true' population mean partial correlation coefficient (PCC). The sample sizes of the primary study's multiple regressions are the same as reported in Tables 1 and 2. **Bias** is the difference between the meta-analysis estimate and ρ calculated from 50 estimated partial correlation coefficients and averaged across 10,000 replications. **RMSE** is the square root of the mean squared error. ***Coverage*** is the proportion of 10,000 meta-analysis 95% confidence intervals that contain ρ. **RE$_{ss}$** is the random-effect's estimate of the mean using S_2^2, from eq. (4) and the small-sample adjustment (n-2)/(n-1). **UWLS$_{+3}$** is the unrestricted weighted least squares' estimate of the mean using S_2^2 from eq. (4) and df_{+3} as the degrees of freedom in PCC's formulae. **RE$_z$** is the random-effect's estimate of Fisher's z converted back to PCC. [a]Average biases are averages across the absolute values of the biases. Biases reported as '.0000' are < |$\pm$.00005|.