# Meta-analyses of partial correlations are biased: Detection and solutions

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**Abstract**

We demonstrate that all meta-analyses of partial correlations are biased, and yet hundreds of meta-analyses of partial correlation coefficients (PCC) are conducted each year widely across economics, business, education, psychology, and medical research. To address these biases, we offer a new weighted average, UWLS+3. UWLS+3 is the unrestricted weighted least squares weighted average that makes an adjustment to the degrees of freedom that are used to calculate partial correlations and, by doing so, renders trivial any remaining meta-analysis bias. Our simulations also reveal that these meta-analysis biases are small-sample biases ($n$ < 200), and a simple correction factor of *(n-2)/(n-1)* greatly reduces these small-sample biases. In many applications where primary studies typically have hundreds or more observations, partial correlations can be meta-analyzed in standard ways with only negligible bias. However, in other fields in the social and the medical sciences that are dominated by small samples, these meta-analysis biases are easily avoidable by our proposed methods.

**Keywords**: partial correlation coefficients, meta-analysis, bias, small sample

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## 1. INTRODUCTION

Every year, hundreds of meta-analyses of partial correlation coefficients (PCC) are conducted widely across economics, business, education, psychology, and medical research.[i] Some researchers consider partial correlations to be the preferred effect size to summarize multiple regressions.[1] Others recommend using partial correlations as a last resort when different measures of the dependent variable and/or the independent variable of interest are routinely employed in the relevant area of research.[2] What is not widely recognized is that all meta-analyses of PCCs are biased regardless of whether fixed effect (FE), random effects (RE), or the unrestricted weighted least squares (UWLS) weighted average are employed and in the absence of any publication selection bias.[ii] The purpose of this paper is to offer a simple and practical solution to these meta-analysis biases.

## 2. PARTIAL CORRELATION COEFFICIENTS

Across many disciplines, multiple regressions are employed to evaluate the effect of a treatment, condition, or variable upon some outcome of interest after controlling for other, potential contaminating, effects or obscuring complexities. Multiple regression can be represented as:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots + \beta_j X_{ji} + \varepsilon_i \qquad i = 1,2,\ldots,n, \tag{1}$$

where $Y$ is the dependent variable or outcome of interest. Without loss of generalization, we take $X_1$ as the primary variable of interest (perhaps a dichotomous variable representing treatment). The

[i] According to Google Scholar, 229 articles were published in 2022 that include all the following phrases: “partial correlation”, “meta-analysis”, and “publication bias”. Because publication bias is discussed primarily in a meta-analysis context, the last phrase is included to increase the probability that the corresponding study is a meta-analysis, not a primary study citing a meta-analysis. In addition, Google Scholar lists 51 articles published in 2022 that are classified as review articles, include the exact phrases “partial correlation” and “meta-analysis”, but exclude “publication bias”. Because many meta-analyses are not classified as review articles in Google Scholar, we believe that 250 is a lower bound for the number of meta-analyses of PCCs in the Google Scholar database. However, Google Scholar will also list duplicates as theses and preprints may also be listed as published papers. Nonetheless, there are probably at least 200 meta-analyses of partial correlations conducted per year.

[ii] The unrestricted weighted least squares (UWLS) weighted average has been shown to have better statistical properties than RE when there is publication selection bias or when heterogeneity is correlated with sample size (or SE), which meta-research evidence finds in psychology.[11-13] Recently, UWLS is shown to better represent medical research than RE across over 67,000 meta-analyses of approximately 600,000 studies.[18]

other $X$s are independent variables that are thought to affect the outcome. $j$ is the total number of independent variables, and $\varepsilon_i$ represent sampling errors and other residuals.

Multiple regression is used with observational data, quasi-experiments, and other experimental designs when additional experimental conditions or subject characteristics need to be considered. For our purposes, the strength of the experimental design is not relevant as long as the focus of the meta-analysis is upon the estimated multiple regression coefficient, $\hat{\beta}_1$, across the research literature. However, in some cases, observational multiple regressions can offer strong research designs.[3]

The partial regression coefficient, $\hat{\beta}_1$, is not a standardized effect. It is measured in terms of $Y$ per unit increase in $X_1$. Any change in the measure, metric, or scale of either $X_1$ or $Y$ from one study to the next will render different estimates of $\hat{\beta}_1$ uncomparable. PCCs solve this problem. They have the same statistical properties and interpretation as simple bivariate correlations after the effects of $X_2, X_3, \ldots, X_j$ have been eliminated.[4] Simple bivariate Pearson correlations are often employed as effect sizes in meta-analysis, and partial correlations come with the same advantages and limitations.

Gustafson[5] mathematically derived a convenient formula that converts any partial regression coefficient, $\hat{\beta}_1$, into a partial correlation coefficient, $r_p$:

$$r_p = t \Big/ \sqrt{t^2 + df}\,, \qquad (2)$$

where $t = \frac{\hat{\beta}_1}{s_{\hat{\beta}_1}}$ is the conventional $t$-test for the statistical significance of $X_1$ in the explanation of $Y$, and $df = n - j - 1$ are the degrees of freedom available to the multiple regression, eq. (1). $r_p$ can be interpreted as a standardized regression coefficient that estimates the number of standard deviations that $Y$ increases when $X_1$ increases by a one standard deviation, holding all other variables constant, and $r_p^2$ is the proportion of the variation in $Y$ attributable to variation in $X_1$ after eliminating the effects of $X_2, X_3, \ldots, X_j$. Because economics, business, and social sciences, in general, often use different scales and measures of $Y$ and/or $X_1$, PCCs are frequently employed in the meta-analysis of these fields.[2,6,7]

The variance of $r_p$ is:

$$S_1^2 = {\left(1 - r_p^2\right)^2}\Big/{df} \tag{3}$$

Olkin and Siotani[8].[1,9] However, the test of PCC's statistical significance, $H_0$: $\rho$ = 0, requires a slightly different formula for the variance of $r_p$:

$$S_2^2 = {\left(1 - r_p^2\right)}\Big/{df} \tag{4}$$

.[5] These two formulae only differ in that the numerator of $S_2^2$ is not squared, in contrast to the numerator of $S_1^2$. Since, by definition, $-1 \leq r_p \leq 1$, it follows that $S_1^2 < S_2^2$ for all $|r_p| \neq \{0 \text{ or } 1\}$. Using $S_2^2$ and $r_p$ reproduces the *t*-value and the *p*-value of the original estimated partial regression coefficient, $\hat{\beta}_1$; $S_1^2$ does not.

Below we demonstrate that all meta-analyses of PCCs are biased (including FE, RE, and UWLS) regardless of which formula of variance is used. Nevertheless, conventional meta-analyses that use $S_1^2$ cause the estimates of mean effect to be twice as biased as those which employ $S_2^2$. To address these biases, we offer a simple modification to the transformation formula, eq. (2), and a small-sample bias correction for degrees of freedom. First, however, we establish and discuss the bias of the conventional meta-analysis of PCCs. It is only through understanding these biases that a solution can be found.

## 3. META-ANALYSIS BIAS

### 3.1 Simulations

To investigate the statistical properties of the meta-analysis of partial correlations, we conduct Monte Carlo simulations of RE and UWLS estimates of the mean PCC from randomly generated data, which is used to estimate multiple regressions and transform each $\hat{\beta}_1$ to a PCC. Simulations offer an important advantage over other approaches in that we can set the 'true' population value of the PCC, $\rho$, by forcing its value upon the data generating process.

To obtain estimated PCCs for the effect size corresponding to the variable, $X_1$, we start with the following multiple regression:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i \qquad i = 1,2,\ldots,n \tag{5}$$

For simplicity, we set all betas to 1 and assume that $X_{1i}, X_{2i}$, and $\varepsilon_i$ are independently and identically distributed as N(0,1).[iii] The variable, $Y_i$, is generated by eq. (5) after random and independent values are generated for $X_{1i}, X_{2i}$, and $\varepsilon_i$. As a next step, we estimate a multiple regression for eq. (5) and calculate the *t*-value of the estimated regression coefficient $\beta_1$. We then convert $X_1's$ *t*-value to a PCC via eq. (2).

Due to the clarity and simplicity of these data generating processes, the population variance of $Y_i$ *not* attributed to the remaining independent variables, $X_{2i}$, equals 2 because this variance can be computed as the sum of the variances of $X_{1i}$ and $\varepsilon_i$, each of which is set to have variance 1. Both $X_{1i}$ and $\varepsilon_i$ are *independently* distributed with variance 1; hence, this total variance is the sum of $X_{1i}$ and $\varepsilon_i$ variances. Thus, the ratio of $Y_i's$ remaining variance explained by $X_{1i}$ is ½, leading to $\rho = \sqrt{½}$ or 0.707107. This result also follows from Gustafson[5] where $r_p^2$ is shown to be: $\hat{\beta}_1^2 \Big/ \left(\hat{\beta}_1^2 + df \cdot S^2_{\hat{\beta}_1}\right)$. Recall that $\beta_1$ is set to 1, $S^2_{\hat{\beta}_1} = (\sigma^2 / df \cdot \sigma^2_{X_1})$,[10] and both $\sigma^2$ and $\sigma^2_{X_1}$ are set to 1 by design; thus, again $\rho^2$ = ½. In other simulation experiments, we set $\rho$ equal to a 'medium' effect size ($\rho$ = sqrt(.1) = .3162) by dividing $X_{1i}' s$ randomly generated N(0,1) by 3 and a 'small' effect size ($\rho$ = sqrt(1/82) = .1104) by dividing by 9. Doing so makes $X_{1i}'$s variance equal to 1/9 and 1/81, respectively while leaving the error variance at 1—see Table 1.

For each study in our simulations, all the data in eq. (5) is randomly generated, the multiple regression, eq. (5), and its coefficients are estimated, and $r_p$ is calculated from eq. (2). $S_1^2$ is then calculated from eq. (3) and $S_2^2$ from eq. (4), and all these calculations are repeated 50 times to

[iii] We also simulate more complex multiple regression with 4, 6, and 10 independent variables. Results from these more complex multiple regressions are practically equivalent and are reported below and in the Supplement.

represent one meta-analysis.[iv] For each meta-analysis of 50 estimated PCCs, the RE and the UWLS weighted averages are calculated in two ways by using $S_1^2$ and $S_2^2$.

UWLS estimates the simple meta-regression coefficient, $\alpha_1$, from:

$$t_k = \frac{r_{pk}}{SE_k} = \alpha_1 \left(\frac{1}{SE_k}\right) + u_k \qquad k = 1, 2, \ldots, 50 \tag{6}$$

$SE_k$ is calculated as the square root of either $S_1^2$ or $S_2^2$ from their respective formulae above. Any common statistical software automatically calculates UWLS, $\hat{\alpha}_1$ , its standard error, test statistic, and confidence intervals. UWLS and the fixed effect (FE) must have identical point estimates, but UWLS automatically adjusts its standard errors and confidence intervals for heterogeneity when present.[11,12] Because the bias and the square root of the mean square error (RMSE) must be the same for FE and UWLS, we report only UWLS below. Previous simulations have shown that UWLS is statistically superior to RE if there is selection for statistical significance or if small studies are more heterogeneous than larger studies.[11,13] In other cases where RE's model is imposed upon the simulations, the differences between UWLS' and RE's statistical properties are negligible. For each randomly generated meta-analysis, the bias, RMSE and confidence intervals of RE and UWLS are calculated and then averaged across 10,000 replications of all these steps. See the Supplement for the simulation code.

Table 1 reports the results of these simulations using both versions of PCC's variance—eq. (3) and eq. (4). $S_1^2$ consistently produces twice the bias as $S_2^2$ (see also Stanley and Doucouliagos[14] for details on this finding). Table 1 also shows that $S_1^2$ generates larger root mean squared errors and worse coverage (*i.e.,* coverage rates that are often much different than their nominal 95% level) than $S_2^2$. In Section 3.2, below, we discuss the reason for these biases and why $S_1^2$ produces predictably larger biases. These results confirm Stanley and Doucouliagos'[14] finding that the 'correct' variance, $S_1^2$, eq. (3), is not useful in practice when conducting meta-analyses of partial correlations.

[iv] These biases are largely independent of the number of PCCs (*k*) in the meta-analysis. However, the sample size (*n*) of the primary study used to calculate the PCC is a very important determinant of bias. We used other values of *k* and found that meta-analyses of 10 or fewer studies consistently have slightly smaller biases while those with a larger number of estimates (*k* = 200) have slightly larger biases.

### 3.2 Reducing meta-analysis bias to triviality

Looking closely at the biases identified through simulations reveals two additional lessons. First, although these biases are of a notable magnitude for small samples ($n < 50$), all these biases are mere rounding errors (*i.e.*, .005) or smaller for large samples (*i.e.*, $n \geq 200$ or $n \geq 100$ if $S_2^2$ is used). Second, biases consistently halve as $n$ doubles. Figure 1 graphs RE's and UWLS' biases against the inverse of degrees of freedom (1/*df*) when $\rho = \sqrt{½}$, using 10,000 replications of each sample size, $n$ = {10, 20, 40, 80, 160, 320, 640, 1280 & 25, 50, 100, 200, 400, 800, 1600, 2500}. Figure 1 reveals that $S_2^2$ approximately halves RE's bias and that doubling the sample size of the original study halves the bias of each again.

To be more precise, the biases of UWLS with inverse $S_2^2$ weights are a near exact function of the inverse of degrees of freedom (1/*df*):

$$Bias_i = .000069 + .508\left(\frac{1}{df_i}\right) \tag{7}$$

$$t \quad = \quad (1.67) \qquad (505.8) \qquad\qquad ; \; R^2 = .9999453$$

The inverse of degrees of freedom, $\left(\frac{1}{df_i}\right)$, explains over 99.99% of the bias of UWLS ($R^2 \approx$ 99.995%) leaving a 95% margin of error of .0003. Through numerical analysis, we know that the bias of the meta-analysis of PCCs is a function of *df*, and that any remaining error is negligible.

A century ago, Fisher[4] observed that the: "sampling distribution of the partial correlation obtained from $n$ pairs of values, when one variable is eliminated, is the same as the random sampling distribution of a total correlation derived from ($n$-1) pairs. By mere repetition of the above reasoning it appears that when $s$ variates are eliminated the effective size of the sample is diminished to ($n$-$s$)" (p. 330). This suggests that fine-tuning the degrees of freedom in PCC's transformation formula may reduce or practically eliminate this bias. Further simulations confirm that this is indeed the case.

Following Fisher's observation, consider the simple bivariate correlation:

$$r = \frac{S_{xy}}{S_x \cdot S_y} = \sum(X_i - \bar{X})(Y_i - \bar{Y}) \Big/ \sqrt{\sum(X_i - \bar{X})^2} \cdot \sqrt{\sum(Y_i - \bar{Y})^2} \; . \tag{8}$$

The sample covariance, $S_{xy}$, has degrees of freedom (n-2), because two parameters, $\mu_x$ and $\mu_y$, must be first estimated from a sample of n pairs of observations. Each sample variance, $S_x^2$ and $S_y^2$, has (n-1) has degrees of freedom; thus, the denominator is (n-1). This suggests that a correction for degrees of freedom, (n-2)/(n-1), might reduce the small-sample bias of meta-analysis weighted averages that is revealed in Table 1. When the small-sample bias is proportional to 1/*df* and *df* = (n-1) multiplying by (n-2)/(n-1) would correct this small-sample bias. Table 2 reports the random-effects, small-sample correction, $RE_{ss}$, where each sample PCC is first multiplied by (n-2)/(n-1) before the usual random-effects formulae are applied. $RE_{ss}$ greatly reduces the small-sample biases—see Table 1.

These small-sample corrections of PCCs, however, *should not* be interpreted as estimates of individual PCCs. It is widely known that individual correlation estimates, and PCCs, are biased downward (e.g., Olkin and Pratt[15]). Applying this small-sample adjustment would then only make a small downward bias worse. We propose reporting this small-correction correction, (n-2)/(n-1), only for meta-analysis weighted averages while ignoring this small-sample adjustment correction of the individual PCCs.

Table 2 also reports the statistical properties of a new meta-analysis weighted average, $UWLS_{+3}$, that reduces bias to scientific negligibility. $UWLS_{+3}$ uses the same simulation design as before but substituting degrees of freedom that are three larger than the multiple regression's degrees of freedom into PCC's transformation formula, eq. (2). That is, we first calculate PCC as:

$$r_p = t \Big/ \sqrt{t^2 + df_{+3}} \tag{9}$$

for $df_{+3} = n - s + 1$ with *s* as the number of independent variables in the multiple regression held constant in the calculation of the partial correlation of interest (*i.e.*, $s = j - 1$). As displayed in Table 2, $UWLS_{+3}$ eliminates all biases to within < $\pm$.001, and its average absolute bias is only .0002. Table 2 assumes that either there are two independent variables in the multiple regression ($j = 2$) or four (*i.e.*, $j = 4$). To ensure broader generalizability, Supplement Table S1 reports the same simulation design as Table 2, except *j* = 6 &10. Induction suggests that if you can prove trivial bias for one (*i.e.*, *s* =1; Table 2) and trivial bias for some random *s* (*e.g.*, s = 3), then trivial biases generalize to any *s* (*e.g., s* = {5, 9}, Table S1). As a further corroboration of the effective elimination of meta-analysis bias, Table S2 reports the same simulation design but with different values of the population partial correlation coefficient, $\rho$ ={.9487; .2425; 0}.

Now that we have found ways to reduce these biases to scientific triviality, what causes these biases of the conventional meta-analysis of partial correlations? The simple answer is that both formulas for the variance of PCCs are themselves a function of the PCC. Because the weights of meta-analysis are a strictly increasing function of $r_p^2$, it follows that for all $r_p^2 \neq$ {0 or 1} positive sampling errors are assigned more influence in pinning down the meta-analysis estimate compared to negative sampling errors of the same magnitude. In all meta-analyses that use inverse variance weights, based on either $S_1^2$ or $S_2^2$, an upwards bias in magnitude will arise: the absolute expected value delivered by the meta-analysis will surpass $|\rho|$ if the true correlation is not 0 or 1.

Let us assume, for instance, that $\rho$ = 0.7 and examine how estimates with errors of the same magnitude but different signs ( $\pm$ 0.2) are weighted in meta-analysis. For $S_2^2$, an UWLS estimate with a sampling error of +0.2 is assigned a weight proportional to 1/.19 = 5.26, in stark contrast to 1/.75 = 1.333 for a -0.2 sampling error. Here estimates with positive errors are assigned nearly 4 times more influence than estimates with negative errors but equal in size. Few sampling errors will in practice be as large as $\pm$ 0.2, but the aforementioned principle of asymmetric weighting as the root of bias in conventional meta-analysis of partial correlations holds in general: for all sizes of sampling errors and various meta-analysis estimators. Because RE's weights are the inverse of the sampling variance plus a constant ($\tau^2$), this asymmetric weighting of sampling errors is moderated, but not eliminated, by RE. Table 1 shows that RE's biases are somewhat smaller than UWLS', just as we would expect, and these differences are especially clear for small samples when $S_1^2$ is used. Asymmetric weighting of sampling errors biases weighted averages upwards in magnitude. Table 1 confirms these biases.

For bivariate correlations, this issue that the variance is a function of the effect size and that this may be problematic for meta-analysis is widely known. A solution is to convert correlations to Fisher z's, calculate the meta-analysis estimate of the mean and its related statistics, then convert these terms of Fisher zs back to correlations for the purpose of interpretation.[16] As Fisher[4] noted, what is true for correlations is true for partial correlations after degrees of freedom are adjusted for the number of variables eliminated, *s*. Tables 2 and S1 also report the biases, RMSEs, and coverage rates for random effect estimates of Fisher's z that have been converted back to PCCs. Using Fisher's z eliminates most conventional meta-analysis bias. Its biases and MSEs are nearly the same as the simple RE correction for small-sample bias, However, in all cases and by all criteria, $UWLS_{+3}$, has better statistical properties than either Fisher's z or $RE_{ss}$. Although Fisher's z and $RE_{ss}$ produce biases larger than rounding error only for small samples and medium or larger correlations, $UWLS_{+3}$'s bias is still ten times smaller, see Figure 2. Likewise, $UWLS_{+3}$'s RMSEs are smaller, and its coverage rates are closer to the nominal 95% than Fisher's z or $RE_{ss}$. In fact, $RE_{ss}$ CIs are too narrow for large PCCs. Practically speaking, however, all three: Fisher's z, $RE_{ss}$, and $UWLS_{+3}$ solve this problem of biased meta-analyses of partial correlations in the vast majority of cases even though $UWLS_{+3}$ is slightly better.

### 3.3 Heterogeneity

Notable heterogeneity across studies within an area of research is common in all disciplines. In psychology, for example, the observed variance from study-to-study is about 4 times larger than what reported standard errors imply (*i.e.*, median $I^2$ = 74%).[17] To ensure that partial correlation's biases are robust to heterogeneity, we have modified the same simulation design to produce heterogeneity at levels seen in psychology. Tables 3 and 4 report the same simulations as Tables 1 and 2, except that random heterogeneity is added to each study's estimated correlation in each meta-analysis. We first convert each randomly generated estimated correlation to Cohen's d, add a random normal deviation with mean zero and standard deviation {.5, .3, .2*d*} as $\rho$ is: {0.7071, 0. 3162, 0.1104}, and, lastly, transform this back to a partial correlation. We transform to Cohen's d in this way to produce random heterogeneity consistent with the random-effect model and to reproduce roughly the same distribution of heterogeneity as seen in psychology.[v] Table 3 shows that the biases of the meta-analysis of correlations remain, while Table 4 confirms that Fisher's z and the small-sample corrections introduced here consistently reduce these biases to scientific negligibility.

[v] Generating heterogeneity though random variations to $X_1$'s regression coefficient, $\beta_1$=1 $\pm$ N(0, .2) produces approximately same overall results as Table 3 and Table 4.

## 4. DISCUSSION

Meta-analyses of partial correlation coefficients (PCC) are generally biased. We offer new solutions: $UWLS_{+3}$ and the small-sample correction, $RE_{ss}$. Although these biases are ubiquitous, the good news is that they practically and scientifically disappear when the primary studies employ larger samples ($n \geq 200$). Thus, these biases will typically not be a notable factor in the meta-analysis of econometric studies in economics and finance, which often involve hundreds of observations or more.[vi] Nonetheless, for many areas of education, business, psychology, medicine and health, meta-analysts need to use $UWLS_{+3}$, $RE_{ss}$, or Fisher's z in the meta-analysis of PCCs.

An important limitation to our study is that the primary research literatures will typically be much richer than what our simulations have assumed. We abstract from such complexities to isolate and detect these biases and then to understand their underlying cause. However, many meta-analyses will include some studies which may be sufficiently large to have negligible bias, which will likely moderate the weighted averages of these biases. Thus, in most social science applications, it is unlikely that the bias of the meta-analysis of partial correlation coefficients will be as large as those revealed here in small samples.

Both $UWLS_{+3}$ and $RE_{ss}$ are easy to implement. To calculate $UWLS_{+3}$, meta-analysts merely need to add 3 to *df* in PCC's transformation formula, eq. (2), and the formula that calculates PCC's variance, $S_2^2$, eq. (4). $UWLS_{+3}$ is the simple regression coefficient, eq. (6), and it can be estimated using any regression software. Note that UWLS' regression does not have an intercept (or a 'constant'). Aside from small improvements to bias, MSE, and coverage rates over Fisher's z, $UWLS_{+3}$'s advantage lies in its computational simplicity and the clarity of its interpretation.

Unlike the meta-analysis of Fisher's z, $UWLS_{+3}$ is a partial correlation and can be understood entirely as such. Neither $UWLS_{+3}$ nor $RE_{ss}$ need to be transformed back to a correlation to be interpretable. This is particularly helpful for multiple meta-regression analysis (MRA). In economic applications, meta-analyses of PCCs are common and frequently involve a dozen or more moderator variables. To understand the impact of important MRA coefficients, it is necessary to interpret them in terms of the effect size studied, in this case partial correlation coefficients.

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[vi] Across 358 economic meta-analyses about 2/3[rds] of 174,542 estimates are computed from sample sizes larger than 200.[19]

When Fisher's zs are the object of meta-analysis and MRA, it is easy to misinterpret MRA results as correlations. With multiple MRA, the inverse Fisher's z transformation, $\text{PCC} = e^{\left[\frac{2\cdot Z-1}{2\cdot Z+1}\right]}$, would need to be separately employed multiple times if Fisher's zs are meta-analyzed.

Computational simplicity and clarity of interpretation are also advantages of $RE_{ss}$. When there is little or no heterogeneity, Table 2, $UWLS_{+3}$ dominates both Fisher's z and $RE_{ss}$. However, $RE_{ss}$ has a limitation not seen in either $UWLS_{+3}$ or Fisher's z. When the 'true' correlation is very large, $\rho$ = .9487, $RE_{ss}$ has notably larger biases than either $UWLS_{+3}$ or Fisher's z. However, we have not seen average PCCs as large .7 in any economics meta-analysis,[vii] and no bivariate average correlation (RE) is has an absolute value larger than .6 among the 108 *Psychological Bulletin* meta-analyses.[17]

## V. CONCLUSION

We find that all meta-analyses of partial correlations are biased, and we offer simple remedies for these biases, $UWLS_{+3}$ and $RE_{ss}$. Both make a simple adjustment to the degrees of freedom used to calculate partial correlations and thereby render trivial any remaining bias. $UWLS_{+3}$ outperforms $RE_{ss}$ and the more cumbersome application of Fisher's z, but all three reduce bias to trivial magnitudes in the great majority of practical applications. Our simulations also reveal that all biases are small-sample biases ($n$ < 200). Thus, in applications where primary studies typically have hundreds and even more observations, PCCs can be meta-analyzed in any of the above ways without notable bias. However, for many fields in the social and the medical sciences where small-sample studies dominate, these biases are easily avoidable by employing either $UWLS_{+3}$ or $RE_{ss}$.

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[vii] Among 151 meta-analyses of partial correlations for which we have data, the UWLS estimate ranges from -0.45 to 0.55. The median absolute UWLS is 0.021.[19]

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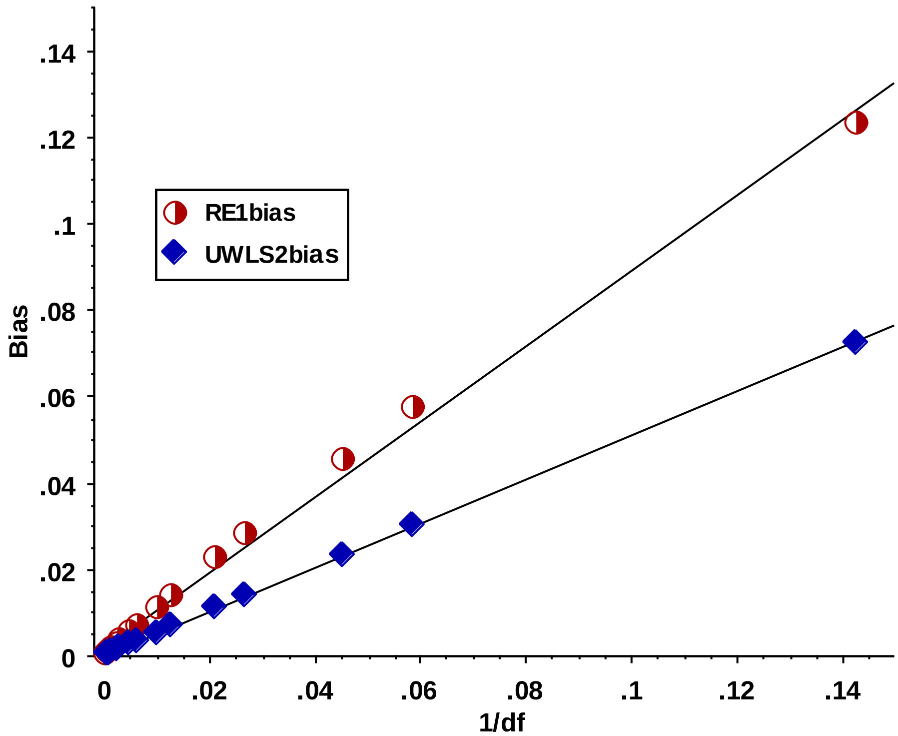


FIGURE 1: Biases of random-effects and the unrestricted weight least square. Each point represents an average bias across 10,000 replications. **RE1bias** is random effects' bias that use PCC variance, $S_1^2$, from eq. (3). **UWLS2bias** is UWLS' bias using $S_2^2$ from eq. (4).

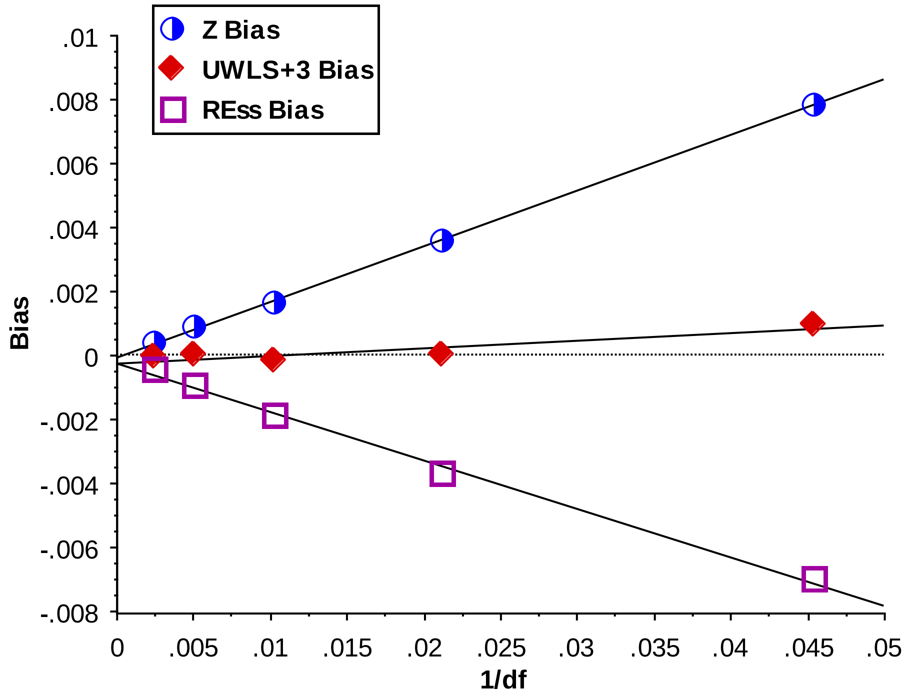


FIGURE 2: Biases of the meta-analysis of Fisher's z converted back to PCC (**Z Bias**), the unrestricted weight least squares with 3 additional degrees of freedom (**UWLS+3**), and the random-effect's estimate of the mean, **RE**$_{ss}$, using $S_2^2$, from eq. (3) and the small-sample adjustment (n-2)/(n-1) for $\rho = \sqrt{½}$ and 10,000 replications. See Table 2 and its discussion.

Table 1: The meta-analyses of PCCs (RE and UWLS) using different formulae for PCC's variance

| Design | | **Bias** | | | | **RMSE** | | | | **Coverage** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho$ | **n** | **RE$_1$** | **RE$_2$** | **UWLS$_1$** | **UWLS$_2$** | **RE$_1$** | **RE$_2$** | **UWLS$_1$** | **UWLS$_2$** | **RE$_1$** | **RE$_2$** | **UWLS$_1$** | **UWLS$_2$** |
| .7071 | 25 | .0455 | .0233 | .0540 | .0233 | .0478 | .0278 | .0568 | .0278 | .1428 | .8521 | .0588 | .3787 |
| .7071 | 50 | .0223 | .0108 | .0254 | .0108 | .0245 | .0149 | .0277 | .0149 | .4103 | .9497 | .2954 | .5928 |
| .7071 | 100 | .0111 | .0053 | .0125 | .0053 | .0131 | .0088 | .0145 | .0088 | .6619 | .9796 | .5788 | .7136 |
| .7071 | 200 | .0055 | .0026 | .0061 | .0026 | .0075 | .0057 | .0080 | .0057 | .8109 | .9878 | .7714 | .7734 |
| .7071 | 400 | .0028 | .0013 | .0031 | .0013 | .0045 | .0038 | .0048 | .0038 | .8824 | .9911 | .8585 | .8025 |
| .3162 | 25 | .0347 | .0173 | .0490 | .0194 | .0461 | .0336 | .0591 | .0348 | .7358 | .8987 | .5843 | .8312 |
| .3162 | 50 | .0179 | .0083 | .0216 | .0089 | .0265 | .0208 | .0295 | .0211 | .8327 | .9329 | .7810 | .8900 |
| .3162 | 100 | .0091 | .0042 | .0104 | .0045 | .0161 | .0138 | .0170 | .0139 | .8892 | .9469 | .8714 | .9118 |
| .3162 | 200 | .0045 | .0020 | .0050 | .0022 | .0102 | .0093 | .0105 | .0093 | .9246 | .9612 | .9127 | .9278 |
| .3162 | 400 | .0022 | .0009 | .0024 | .0010 | .0068 | .0065 | .0069 | .0065 | .9424 | .9599 | .9339 | .9349 |
| .1104 | 25 | .0134 | .0065 | .0198 | .0079 | .0360 | .0321 | .0412 | .0328 | .9114 | .9413 | .8771 | .9234 |
| .1104 | 50 | .0073 | .0034 | .0088 | .0039 | .0225 | .0208 | .0234 | .0210 | .9332 | .9517 | .9246 | .9410 |
| .1104 | 100 | .0034 | .0015 | .0040 | .0017 | .0150 | .0144 | .0152 | .0145 | .9431 | .9532 | .9362 | .9430 |
| .1104 | 200 | .0017 | .0007 | .0019 | .0008 | .0102 | .0100 | .0103 | .0100 | .9495 | .9548 | .9424 | .9468 |
| .1104 | 400 | .0009 | .0005 | .0010 | .0005 | .0071 | .0070 | .0071 | .0070 | .9596 | .9623 | .9533 | .9535 |
| **Average** | | .0122 | .0059 | .0150 | .0063 | .0196 | .0153 | .0221 | .0155 | .7953 | .9482 | .7520 | .8310 |

*Notes*: $\rho$ is the 'true' population mean partial correlation coefficient (PCC). **n** is the sample size used in the primary study's multiple regression. **Bias** is the difference between the meta-analysis estimate and $\rho$ calculated from 50 estimated partial correlation coefficients and averaged across 10,000 replications. **RMSE** is the square root of the mean squared error. ***Coverage*** is the proportion of 10,000 meta-analyses' 95% confidence intervals that contain $\rho$. **RE** is the random-effect's estimate of the mean, and **UWLS** is the unrestricted weighted least squares' estimate of the mean. The subscripts (1 and 2) refer to the use of either the PCC variance, $S_1^2$, from eq. (3) or $S_2^2$ from eq. (4) to calculate the RE and UWLS weighted averages.

Table 2: **$RE_{ss}$**, **$RE_z$**, and **$UWLS_{+3}$** meta-analyses of partial correlations

| **2 IVs: Partial Correlation of $X_1$ from $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i$** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Design | | **Bias** | | | **RMSE** | | | **Coverage** | | |
| $\rho$ | **n** | **$RE_{ss}$** | **$RE_z$** | **$UWLS_{+3}$** | **$RE_{ss}$** | **$RE_z$** | **$UWLS_{+3}$** | **$RE_{ss}$** | **$RE_z$** | **$UWLS_{+3}$** |
| .7071 | 25 | -.0070 | .0078 | .0009 | .0161 | .0168 | .0155 | .9891 | .9281 | .9431 |
| .7071 | 50 | -.0037 | .0036 | .0001 | .0107 | .0109 | .0105 | .9914 | .9460 | .9511 |
| .7071 | 100 | -.0019 | .0017 | -.0001 | .0075 | .0073 | .0072 | .9923 | .9530 | .9514 |
| .7071 | 200 | -.0010 | .0008 | -.0001 | .0051 | .0051 | .0051 | .9938 | .9539 | .9503 |
| .7071 | 400 | -.0004 | .0004 | .0000 | .0035 | .0036 | .0036 | .9953 | .9551 | .9480 |
| .3162 | 25 | .0050 | .0067 | .0008 | .0281 | .0284 | .0275 | .9516 | .9492 | .9408 |
| .3162 | 50 | .0017 | .0032 | .0003 | .0188 | .0190 | .0187 | .9569 | .9519 | .9458 |
| .3162 | 100 | .0008 | .0014 | .0000 | .0129 | .0131 | .0130 | .9626 | .9553 | .9460 |
| .3162 | 200 | .0005 | .0006 | -.0002 | .0091 | .0091 | .0091 | .9646 | .9567 | .9482 |
| .3162 | 400 | .0002 | .0004 | .0000 | .0063 | .0064 | .0064 | .9659 | .9556 | .9497 |
| .1104 | 25 | .0016 | .0024 | .0002 | .0306 | .0306 | .0301 | .9478 | .9545 | .9368 |
| .1104 | 50 | .0007 | .0011 | .0000 | .0208 | .0206 | .0203 | .9496 | .9593 | .9481 |
| .1104 | 100 | .0004 | .0007 | .0001 | .0143 | .0143 | .0142 | .9527 | .9584 | .9489 |
| .1104 | 200 | .0003 | .0002 | -.0001 | .0099 | .0100 | .0100 | .9573 | .9569 | .9485 |
| .1104 | 400 | .0001 | .0001 | -.0001 | .0069 | .0071 | .0070 | .9609 | .9564 | .9495 |
| **Average** | | .0017 | .0021 | .0002[a] | .0134 | .0135 | .0132 | .9688 | .9527 | .9471 |
| **4 IVs: Partial Correlation of $X_1$ from $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \varepsilon_i$** | | | | | | | | | | |
| .7071 | 25 | -.0048 | .0083 | .0009 | .0160 | .0163 | .0164 | .9920 | .9284 | .9424 |
| .7071 | 50 | -.0032 | .0037 | -.0001 | .0108 | .0107 | .0106 | .9930 | .9434 | .9447 |
| .7071 | 100 | -.0017 | .0018 | -.0001 | .0074 | .0073 | .0073 | .9929 | .9513 | .9512 |
| .7071 | 200 | -.0009 | .0008 | -.0001 | .0051 | .0050 | .0050 | .9949 | .9554 | .9506 |
| .7071 | 400 | -.0004 | .0004 | .0000 | .0036 | .0036 | .0036 | .9935 | .9556 | .9490 |
| .3162 | 25 | .0064 | .0063 | .0000 | .0297 | .0289 | .0289 | .9491 | .9520 | .9380 |
| .3162 | 50 | .0020 | .0029 | -.0001 | .0192 | .0191 | .0191 | .9551 | .9545 | .9456 |
| .3162 | 100 | .0008 | .0014 | -.0001 | .0131 | .0129 | .0130 | .9606 | .9588 | .9516 |
| .3162 | 200 | .0005 | .0006 | -.0001 | .0090 | .0091 | .0092 | .9658 | .9592 | .9518 |
| .3162 | 400 | .0002 | .0003 | -.0001 | .0064 | .0063 | .0065 | .9642 | .9591 | .9554 |
| .1104 | 25 | .0025 | .0029 | .0005 | .0325 | .0312 | .0316 | .9440 | .9553 | .9379 |
| .1104 | 50 | .0010 | .0012 | .0000 | .0212 | .0209 | .0209 | .9508 | .9580 | .9463 |
| .1104 | 100 | .0004 | .0007 | .0001 | .0145 | .0144 | .0145 | .9548 | .9553 | .9473 |
| .1104 | 200 | .0001 | .0002 | -.0001 | .0102 | .0100 | .0101 | .9508 | .9562 | .9472 |
| .1104 | 400 | -.0001 | .0001 | .0000 | .0070 | .0071 | .0071 | .9597 | .9543 | .9458 |
| **Average** | | .0017[a] | .0021 | .0002[a] | .0137 | .0138 | .0135 | .9681 | .9531 | .9470 |

*Notes*: $\rho$ is the 'true' population mean partial correlation coefficient (PCC). **n** is the sample size used in the primary study's multiple regression. **Bias** is the difference between the meta-analysis estimate and $\rho$ calculated from 50 estimated partial correlation coefficients and averaged across 10,000 replications. **RMSE** is the square root of the mean squared error. ***Coverage*** is the proportion of 10,000 meta-analysis 95% confidence intervals that contain $\rho$. **$RE_{ss}$** is the random-effect's estimate of the mean using $S_2^2$, from eq. (3) and the small-sample adjustment (n-2)/(n-1). **$UWLS_{+3}$** is the unrestricted weighted least squares' estimate of the mean using $S_2^2$ from eq. (4) and $df_{+3}$ as the degrees of freedom in PCC's formula. **$RE_z$** is the random-effect's estimate of Fisher's z converted back to PCC. [a]Average biases are averages across the absolute values of the biases. Biases reported as '.0000' are < |±.00005|.

Table 3: The meta-analyses of PCCs (RE and UWLS) using different formulae for PCC's variance and with heterogeneity

| Design | | **Bias** | | | | **RMSE** | | | | **Coverage** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho$ | **I²** | **RE₁** | **RE₂** | **UWLS₁** | **UWLS₂** | **RE₁** | **RE₂** | **UWLS₁** | **UWLS₂** | **RE₁** | **RE₂** | **UWLS₁** | **UWLS₂** |
| .7071 | .369 | .0385 | .0245 | .0710 | .0270 | .0435 | .0317 | .0736 | .0328 | .3931 | .7546 | .0322 | .4151 |
| .7071 | .559 | .0124 | .0068 | .0459 | .0149 | .0214 | .0198 | .0485 | .0216 | .7771 | .8724 | .1362 | .6138 |
| .7071 | .731 | -.0012 | -.0045 | .0347 | .0095 | .0156 | .0168 | .0374 | .0169 | .9018 | .9143 | .2611 | .7180 |
| .7071 | .848 | -.0086 | -.0105 | .0292 | .0069 | .0171 | .0184 | .0320 | .0149 | .8657 | .8746 | .3571 | .7586 |
| .7071 | .920 | -.0125 | -.0136 | .0268 | .0058 | .0190 | .0198 | .0296 | .0140 | .7970 | .8217 | .4035 | .7753 |
| .3162 | .404 | .0241 | .0105 | .0601 | .0209 | .0429 | .0355 | .0715 | .0396 | .8424 | .9134 | .5489 | .8360 |
| .3162 | .516 | .0087 | .0011 | .0343 | .0109 | .0285 | .0266 | .0445 | .0287 | .9099 | .9354 | .7167 | .8845 |
| .3162 | .668 | .0004 | -.0036 | .0232 | .0064 | .0225 | .0225 | .0330 | .0233 | .9396 | .9396 | .8015 | .9116 |
| .3162 | .801 | -.0038 | -.0058 | .0184 | .0045 | .0205 | .0209 | .0279 | .0207 | .9459 | .9404 | .8370 | .9224 |
| .3162 | .890 | -.0061 | -.0071 | .0159 | .0034 | .0202 | .0205 | .0257 | .0198 | .9312 | .9282 | .8543 | .9203 |
| .1104 | .319 | .0108 | .0049 | .0217 | .0079 | .0378 | .0346 | .0457 | .0360 | .9182 | .9334 | .8641 | .9168 |
| .1104 | .363 | .0049 | .0015 | .0108 | .0037 | .0263 | .0251 | .0293 | .0257 | .9332 | .9398 | .9102 | .9343 |
| .1104 | .498 | .0017 | -.0001 | .0063 | .0019 | .0204 | .0200 | .0221 | .0204 | .9336 | .9352 | .9242 | .9342 |
| .1104 | .661 | .0001 | -.0008 | .0044 | .0012 | .0170 | .0169 | .0182 | .0172 | .9447 | .9448 | .9344 | .9415 |
| .1104 | .795 | -.0010 | -.0015 | .0032 | .0006 | .0156 | .0156 | .0165 | .0158 | .9435 | .9410 | .9369 | .9419 |
| **Average** | | .0090[a] | .0065[a] | .0271 | .0084 | .0245 | .0230 | .0370 | .0232 | .8651 | .9059 | .6346 | .8283 |

*Notes*: $\rho$ is the 'true' population mean partial correlation coefficient (PCC). Sample sizes as the same as reported in Tables 1 and 2. $0 \leq \mathbf{I^2} \leq 1$ is a relative measure of heterogeneity. **Bias** is the difference between the meta-analysis estimate and $\rho$ calculated from 50 estimated partial correlation coefficients and averaged across 10,000 replications. **RMSE** is the square root of the mean squared error. ***Coverage*** is the proportion of 10,000 meta-analyses' 95% confidence intervals that contain $\rho$. **RE** is the random-effect's estimate of the mean, and **UWLS** is the unrestricted weighted least squares' estimate of the mean. The subscripts (1 and 2) refer to the use of either the PCC variance, $S_1^2$, from eq. (3) or $S_2^2$ from eq. (4) to calculate the RE and UWLS weighted averages. [a]Average biases are averages across the absolute values of the biases.

Table 4: **RE$_{ss}$**, **RE$_z$**, and **UWLS$_{+3}$** meta-analyses of partial correlations with heterogeneity

| **2 IVs: Partial Correlation of $X_1$ from $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i$** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Design | | **Bias** | | | **RMSE** | | | **Coverage** | | |
| $\rho$ | **I$^2$** | **RE$_{ss}$** | **RE$_z$** | **UWLS$_{+3}$** | **RE$_{ss}$** | **RE$_z$** | **UWLS$_{+3}$** | **RE$_{ss}$** | **RE$_z$** | **UWLS$_{+3}$** |
| .7071 | .369 | -.0058 | .0024 | .0041 | .0199 | .0199 | .0203 | .9614 | .9404 | .9465 |
| .7071 | .559 | -.0068 | -.0016 | .0043 | .0192 | .0165 | .0167 | .9110 | .9429 | .9378 |
| .7071 | .730 | -.0113 | -.0038 | .0043 | .0198 | .0152 | .0149 | .8717 | .9392 | .9397 |
| .7071 | .848 | -.0140 | -.0046 | .0045 | .0205 | .0145 | .0140 | .8233 | .9340 | .9333 |
| .7071 | .919 | -.0154 | -.0053 | .0044 | .0210 | .0144 | .0136 | .7897 | .9279 | .9317 |
| .3162 | .404 | -.0004 | .0037 | .0020 | .0333 | .0327 | .0331 | .9305 | .9421 | .9388 |
| .3162 | .515 | -.0049 | .0001 | .0018 | .0265 | .0256 | .0261 | .9328 | .9470 | .9456 |
| .3162 | .669 | -.0068 | -.0013 | .0022 | .0233 | .0222 | .0226 | .9316 | .9427 | .9447 |
| .3162 | .800 | -.0075 | -.0022 | .0022 | .0215 | .0204 | .0207 | .9274 | .9398 | .9416 |
| .3162 | .890 | -.0077 | -.0025 | .0023 | .0204 | .0190 | .0192 | .9270 | .9430 | .9461 |
| .1104 | .320 | .0012 | .0018 | .0003 | .0326 | .0334 | .0335 | .9413 | .9461 | .9373 |
| .1104 | .364 | -.0006 | .0005 | .0003 | .0245 | .0248 | .0249 | .9405 | .9427 | .9417 |
| .1104 | .500 | -.0006 | .0001 | .0004 | .0193 | .0199 | .0201 | .9460 | .9415 | .9440 |
| .1104 | .661 | -.0010 | -.0001 | .0006 | .0167 | .0170 | .0172 | .9449 | .9445 | .9482 |
| .1104 | .795 | -.0014 | -.0004 | .0004 | .0154 | .0154 | .0155 | .9450 | .9460 | .9506 |
| **Average** | | .0057[a] | .0020[a] | .0023 | .0223 | .0207 | .0208 | .9149 | .9413 | .9418 |
| **4 IVs: Partial Correlation of $X_1$ from $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \varepsilon_i$** | | | | | | | | | | |
| .7071 | .349 | -.0031 | .0033 | .0044 | .0195 | .0206 | .0209 | .9671 | .9372 | .9422 |
| .7071 | .549 | -.0062 | -.0016 | .0042 | .0191 | .0165 | .0167 | .9183 | .9459 | .9430 |
| .7071 | .726 | -.0110 | -.0039 | .0042 | .0195 | .0152 | .0148 | .8738 | .9402 | .9421 |
| .7071 | .847 | -.0139 | -.0049 | .0043 | .0203 | .0147 | .0140 | .8284 | .9331 | .9367 |
| .7071 | .919 | -.0152 | -.0050 | .0048 | .0208 | .0141 | .0135 | .7963 | .9325 | .9326 |
| .3162 | .398 | .0008 | .0048 | .0025 | .0347 | .0338 | .0342 | .9272 | .9461 | .9386 |
| .3162 | .508 | -.0041 | .0005 | .0021 | .0267 | .0259 | .0264 | .9348 | .9440 | .9433 |
| .3162 | .665 | -.0069 | -.0016 | .0018 | .0232 | .0222 | .0225 | .9311 | .9425 | .9439 |
| .3162 | .800 | -.0073 | -.0019 | .0025 | .0213 | .0202 | .0205 | .9323 | .9454 | .9465 |
| .3162 | .889 | -.0081 | -.0023 | .0026 | .0207 | .0192 | .0195 | .9262 | .9413 | .9433 |
| .1104 | .323 | .0012 | .0020 | .0004 | .0344 | .0346 | .0346 | .9392 | .9473 | .9365 |
| .1104 | .358 | -.0001 | .0007 | .0004 | .0247 | .0251 | .0252 | .9410 | .9437 | .9421 |
| .1104 | .495 | -.0010 | .0005 | .0009 | .0199 | .0198 | .0200 | .9392 | .9446 | .9462 |
| .1104 | .658 | -.0011 | -.0005 | .0002 | .0167 | .0171 | .0173 | .9403 | .9390 | .9431 |
| .1104 | .794 | -.0014 | -.0004 | .0005 | .0153 | .0154 | .0156 | .9451 | .9410 | .9457 |
| **Average** | | .0054[a] | .0023[a] | .0024 | .0224 | .0209 | .0210 | .9160 | .9416 | .9417 |

*Notes*: $\rho$ is the 'true' population mean partial correlation coefficient (PCC). The sample sizes of the primary study's multiple regressions are the same as reported in Tables 1 and 2. **Bias** is the difference between the meta-analysis estimate and $\rho$ calculated from 50 estimated partial correlation coefficients and averaged across 10,000 replications. **RMSE** is the square root of the mean squared error. ***Coverage*** is the proportion of 10,000 meta-analysis 95% confidence intervals that contain $\rho$. **RE$_{ss}$** is the random-effect's estimate of the mean using $S_2^2$, from eq. (4) and the small-sample adjustment (n-2)/(n-1). **UWLS$_{+3}$** is the unrestricted weighted least squares' estimate of the mean using $S_2^2$ from eq. (4) and $df_{+3}$ as the degrees of freedom in PCC's formulae. **RE$_z$** is the random-effect's estimate of Fisher's z converted back to PCC. [a]Average biases are averages across the absolute values of the biases. Biases reported as '.0000' are < |$\pm$.00005|.