Lecture 5: Basic Meta-Analysis Tools

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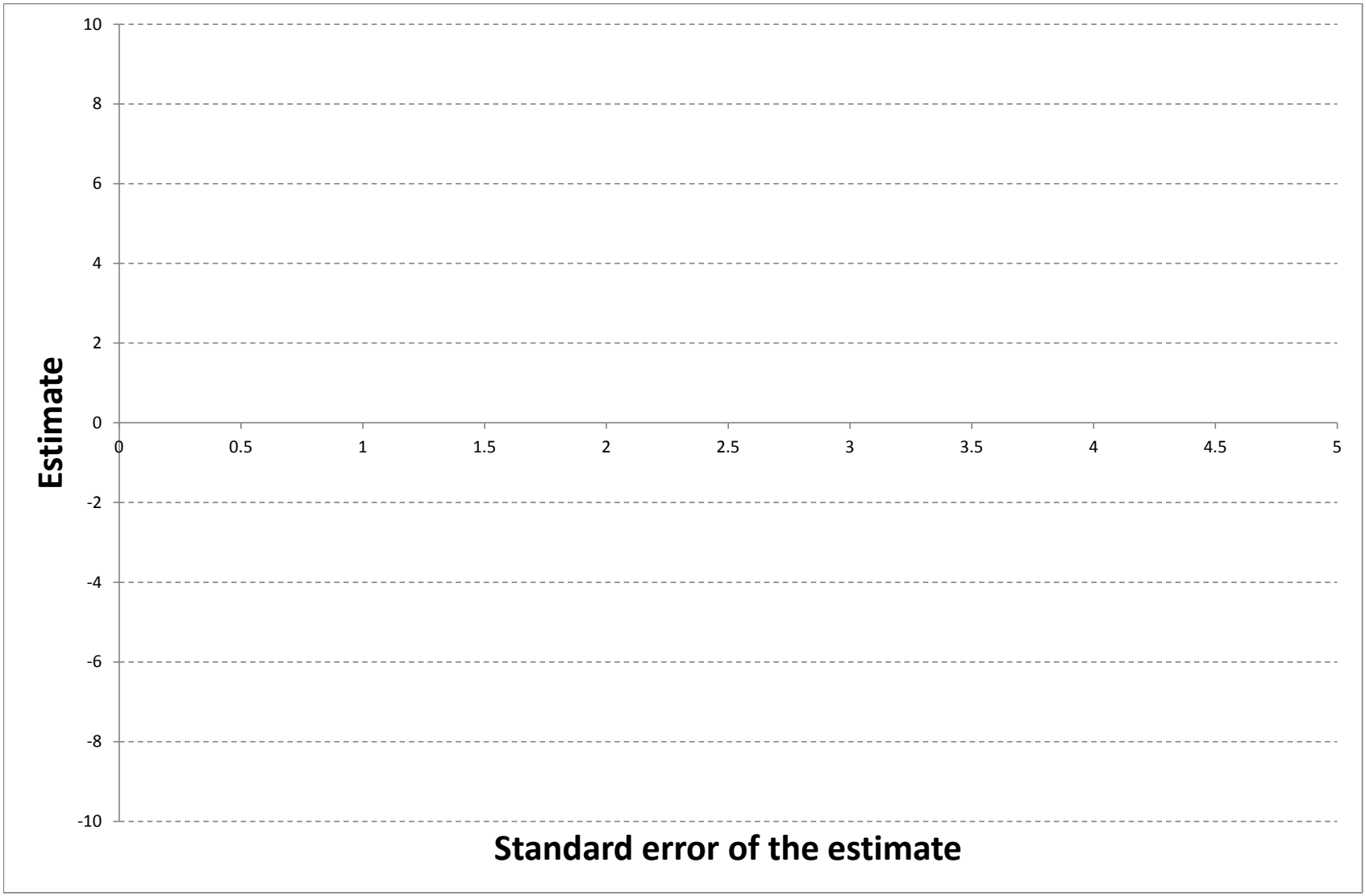
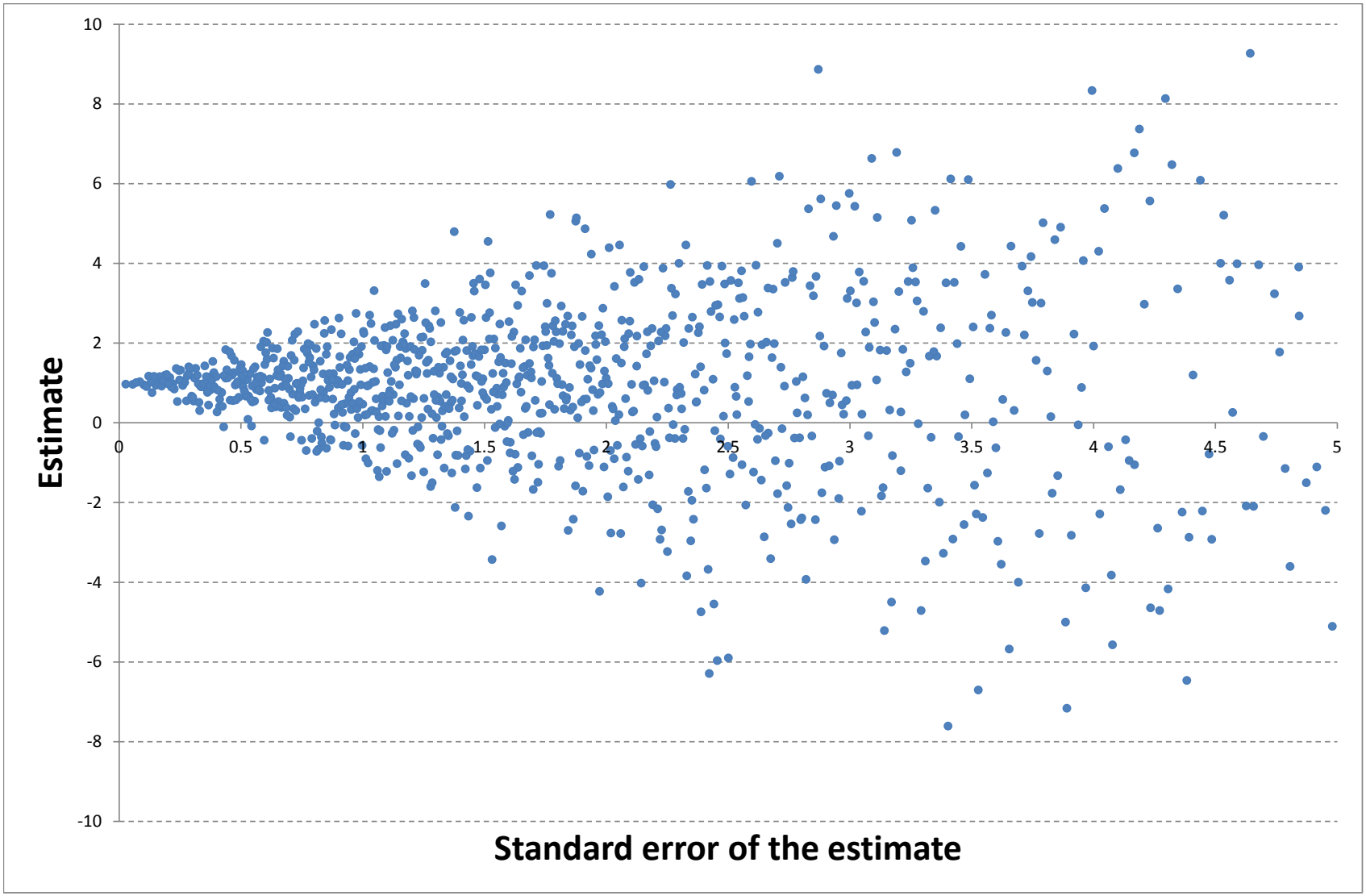
Mean, median, something else?

- Simple mean of reported estimates: often good starting point
- But inefficient, affected by outliers, publication bias
- Median: surprisingly robust to most problems in meta-analysis
- Classical meta-analysis approach: use inverse variance as the weight

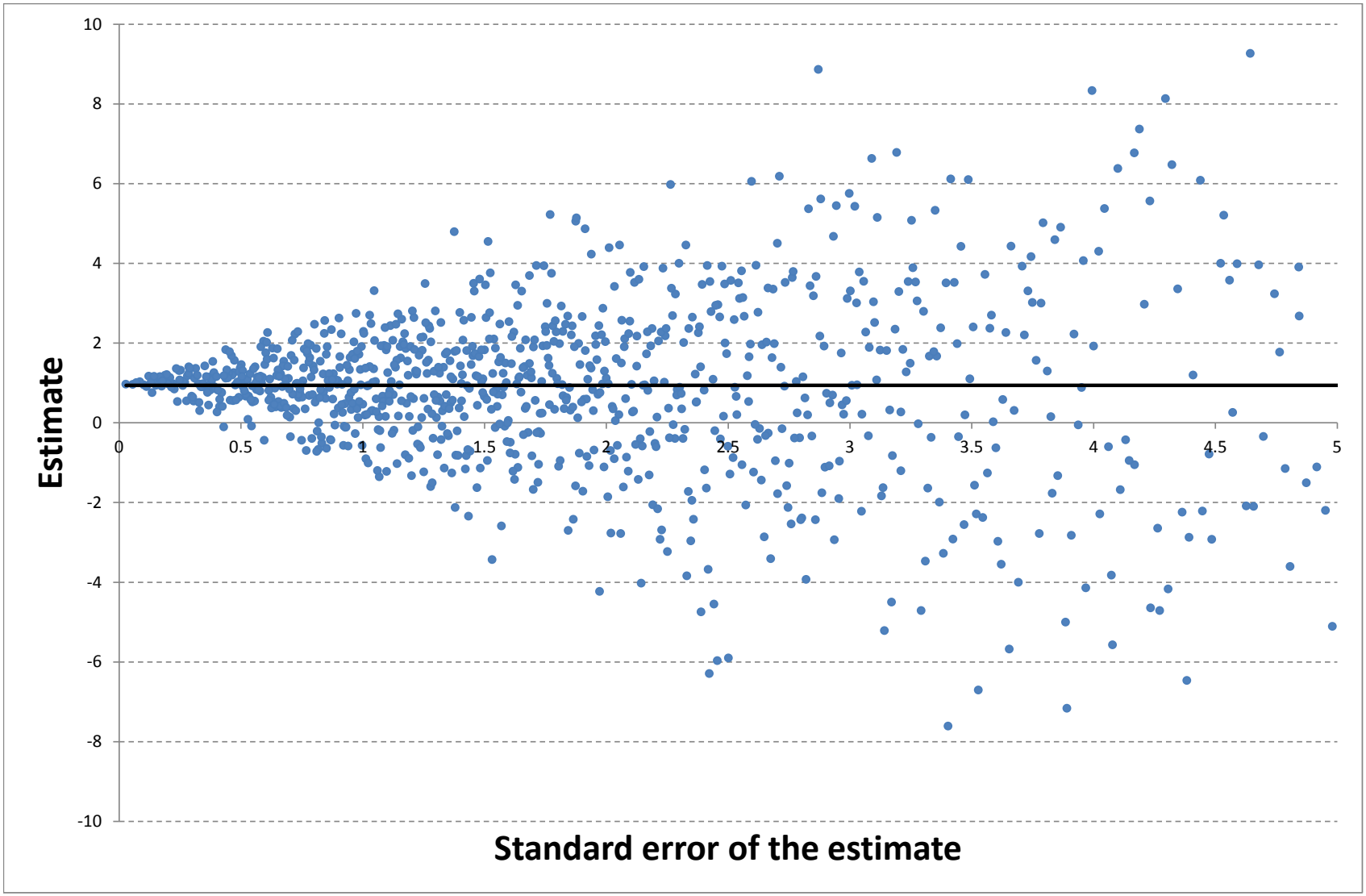
More precision -> more weight



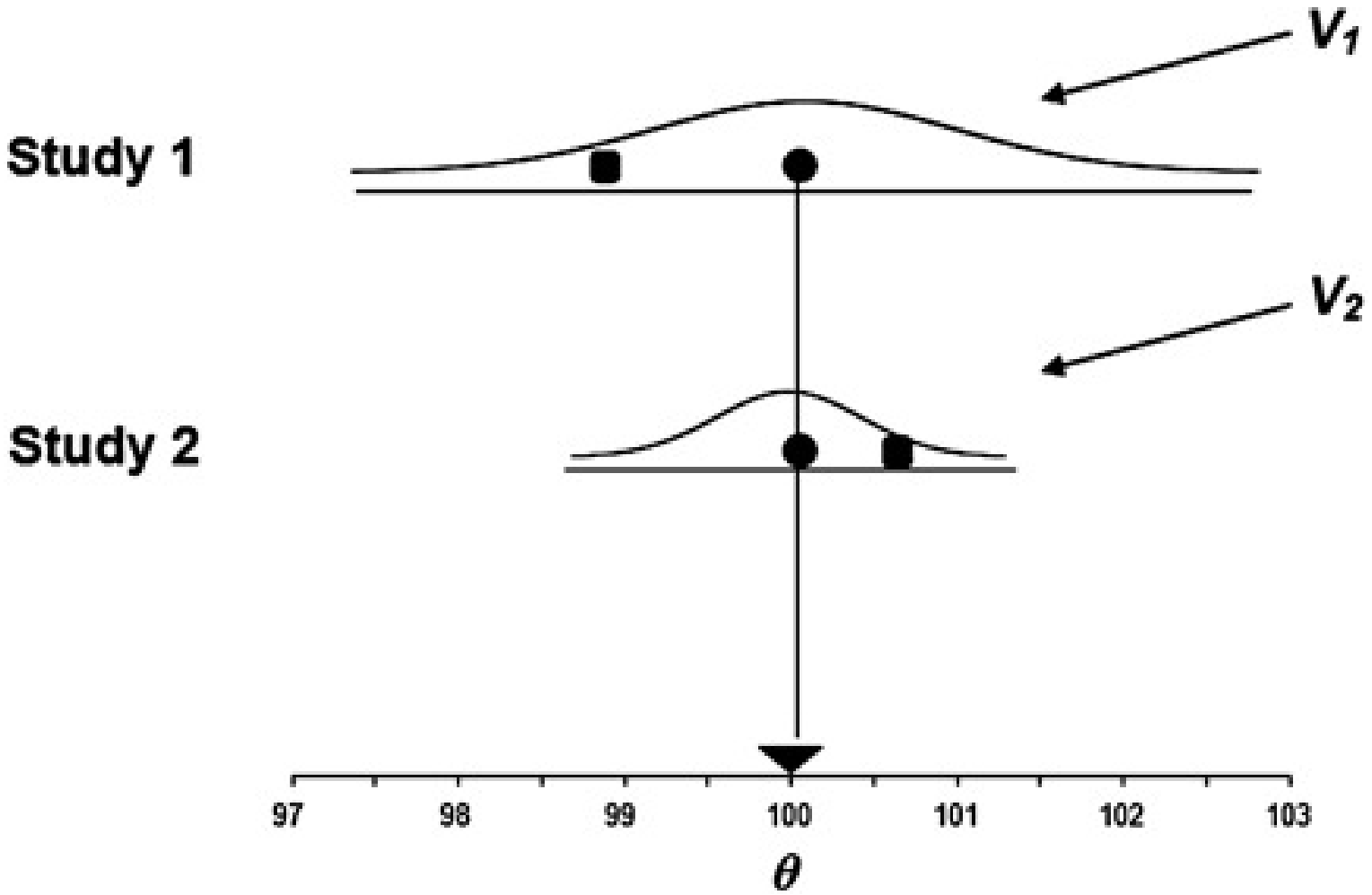
More precision -> more weight



More precision -> more weight



Fixed effect



Fixed effect

$$Y_i = \theta + \varepsilon_i, \tag{1}$$

$$V_i = \frac{\sigma^2}{n}, \tag{2}$$

$$W_i = \frac{1}{V_i}, \tag{3}$$

$$M = \frac{\sum_{i=1}^{k} W_i Y_i}{\sum_{i=1}^{k} W_i}, \tag{4}$$

$$V_M = \frac{1}{\sum_{i=1}^{k} W_i}. \tag{5}$$

FE vs. UWLS: point estimate

- Both methods yield the same point estimate:

$$\hat{\theta} = \frac{\sum w_i \theta_i}{\sum w_i}, \quad w_i = \frac{1}{\sigma_i^2}$$

- Key difference: Variance estimation.

Method	**Point Estimate**
Fixed-Effect (FE)	$\hat{\theta} = \frac{\sum w_i \theta_i}{\sum w_i}$
UWLS	$\hat{\theta} = \frac{\sum w_i \theta_i}{\sum w_i}$

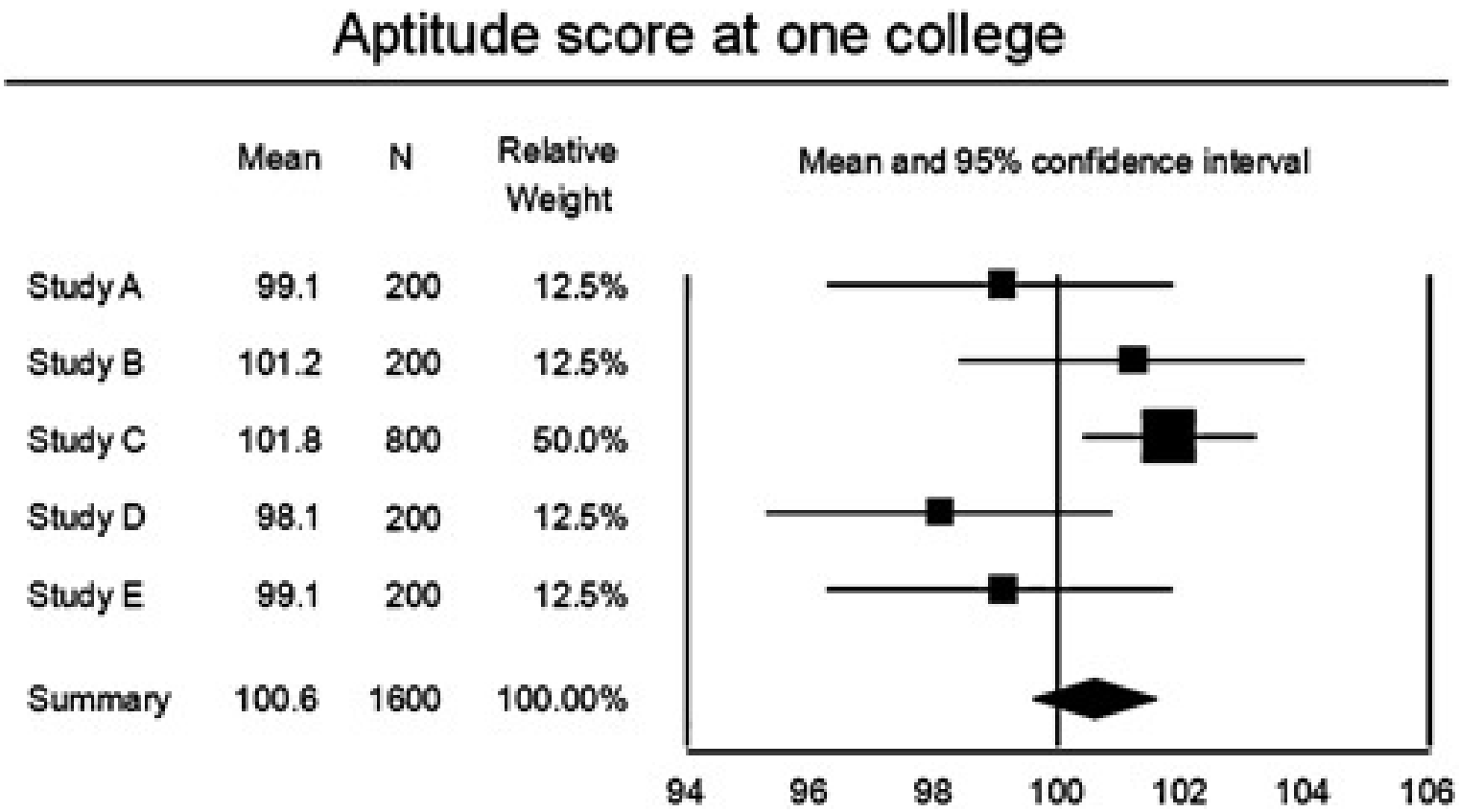
Same estimate, different variance!

FE vs. UWLS: variance

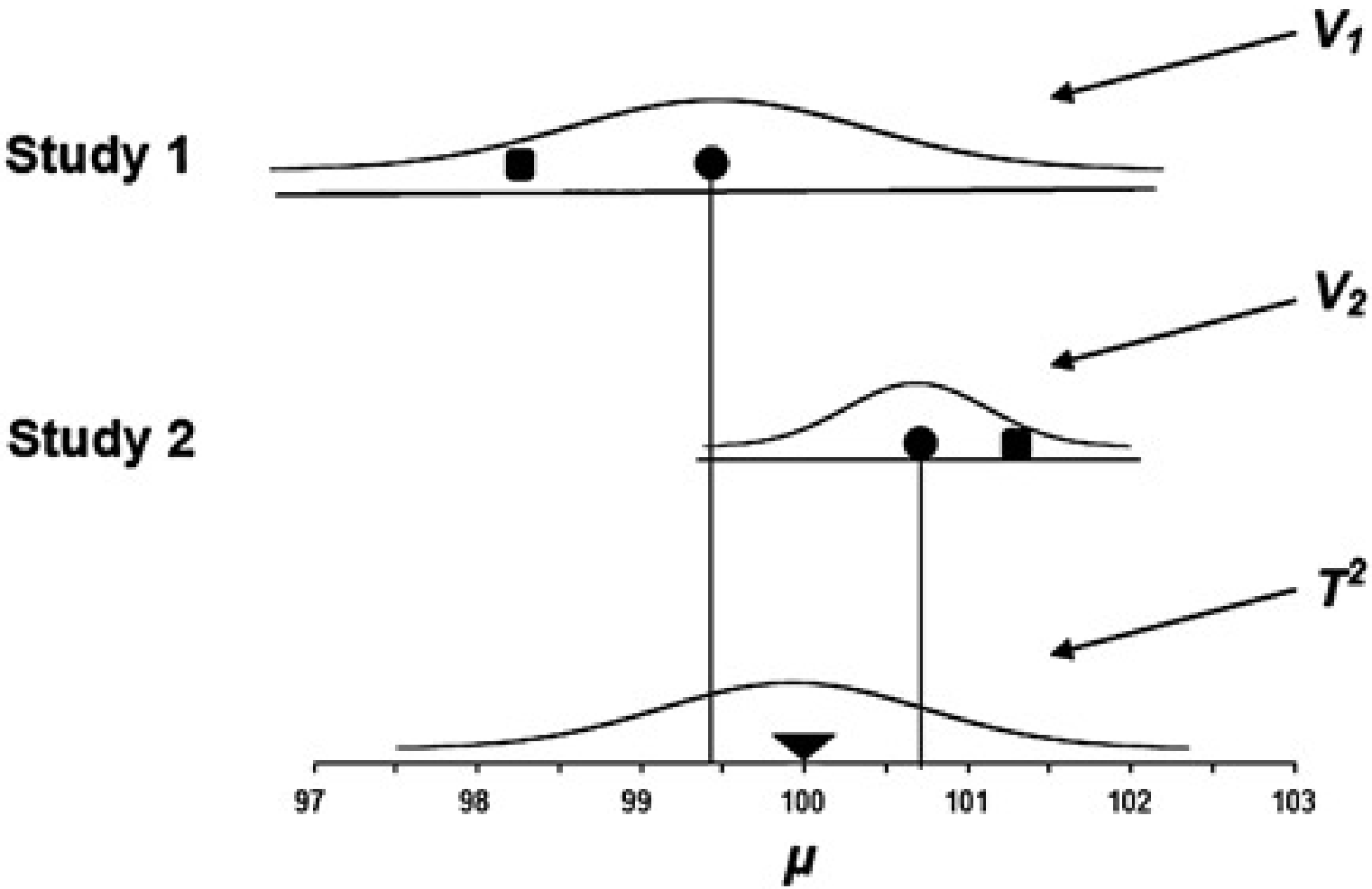
Feature	**Fixed-Effect (FE)**	**UWLS**
Assumption	Identical θ	Allows heterogeneity
Variance formula	$\frac{1}{\sum w_i}$	$\frac{\sum w_i(\theta_i - \hat{\theta})^2}{\sum w_i}$
Variance components	Within-study only	Within + between-study
Robustness	Sensitive to misspecification	More robust

FE underestimates variance if heterogeneity exists!

Fixed effect



Random effects



Random effects

$$Y_i = \mu + \xi_i + \varepsilon_i, \tag{1}$$

$$V_i = \frac{\sigma^2}{n}, \tag{2}$$

$$W_i = \frac{1}{V_i + T^2}, \tag{3}$$

$$M = \frac{\sum_{i=1}^{k} W_i Y_i}{\sum_{i=1}^{k} W_i}, \tag{4}$$

$$V_M = \frac{1}{\sum_{i=1}^{k} W_i}. \tag{5}$$

Estimating τ^2 in random-effects model

Between-study variance τ^2 accounts for heterogeneity across studies. Common estimators:

- **DerSimonian-Laird (DL)**:

$$\tau^2_{DL} = \frac{Q - (k-1)}{C}, \quad C = \sum w_i - \frac{\sum w_i^2}{\sum w_i}$$

  - Simple, but underestimates τ^2 when k is small.

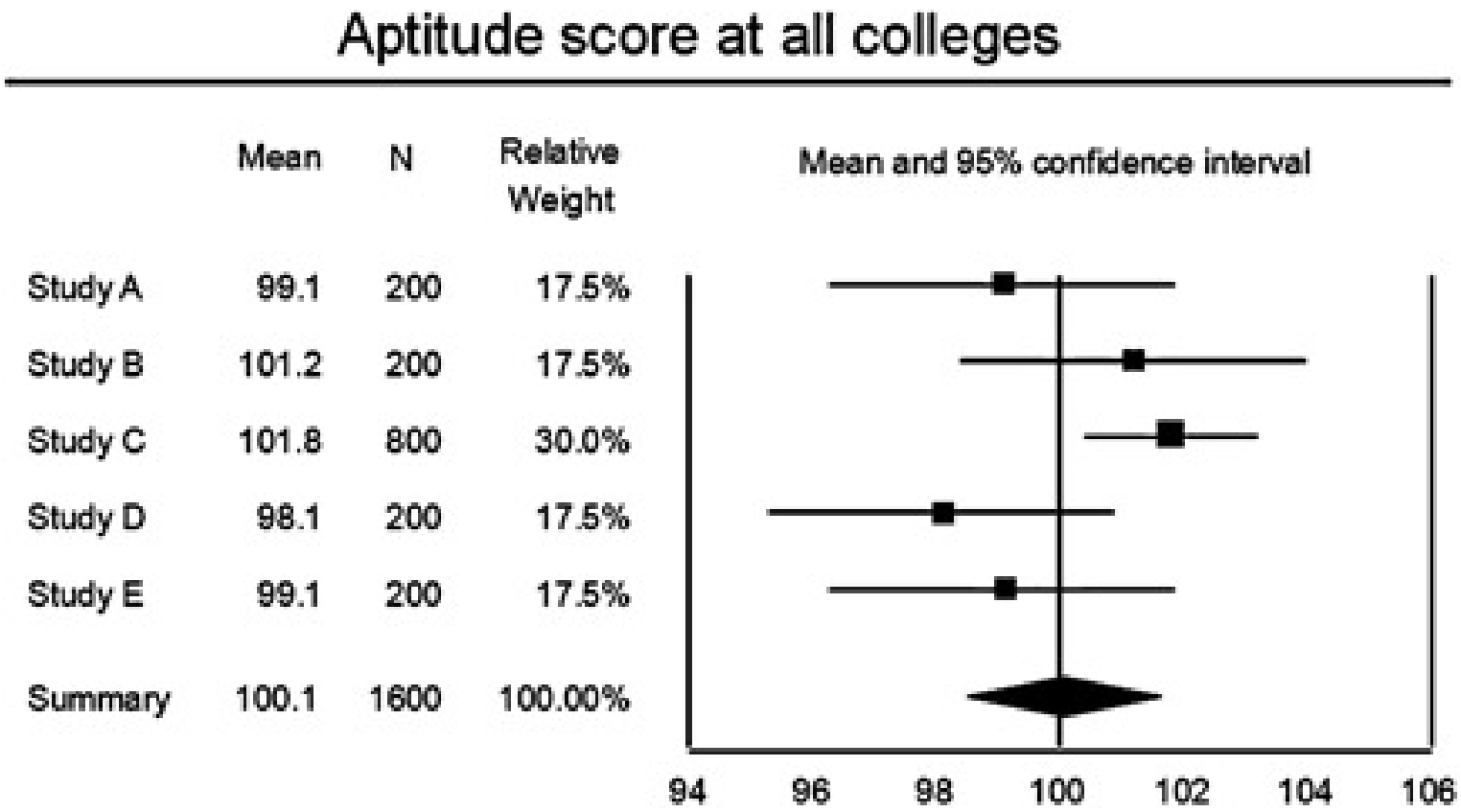
- **Restricted Maximum Likelihood (REML)**:

$$\max_{\tau^2} \sum \log(w_i^* + \tau^2) + \sum \frac{(y_i - \hat{\theta})^2}{w_i^* + \tau^2}$$

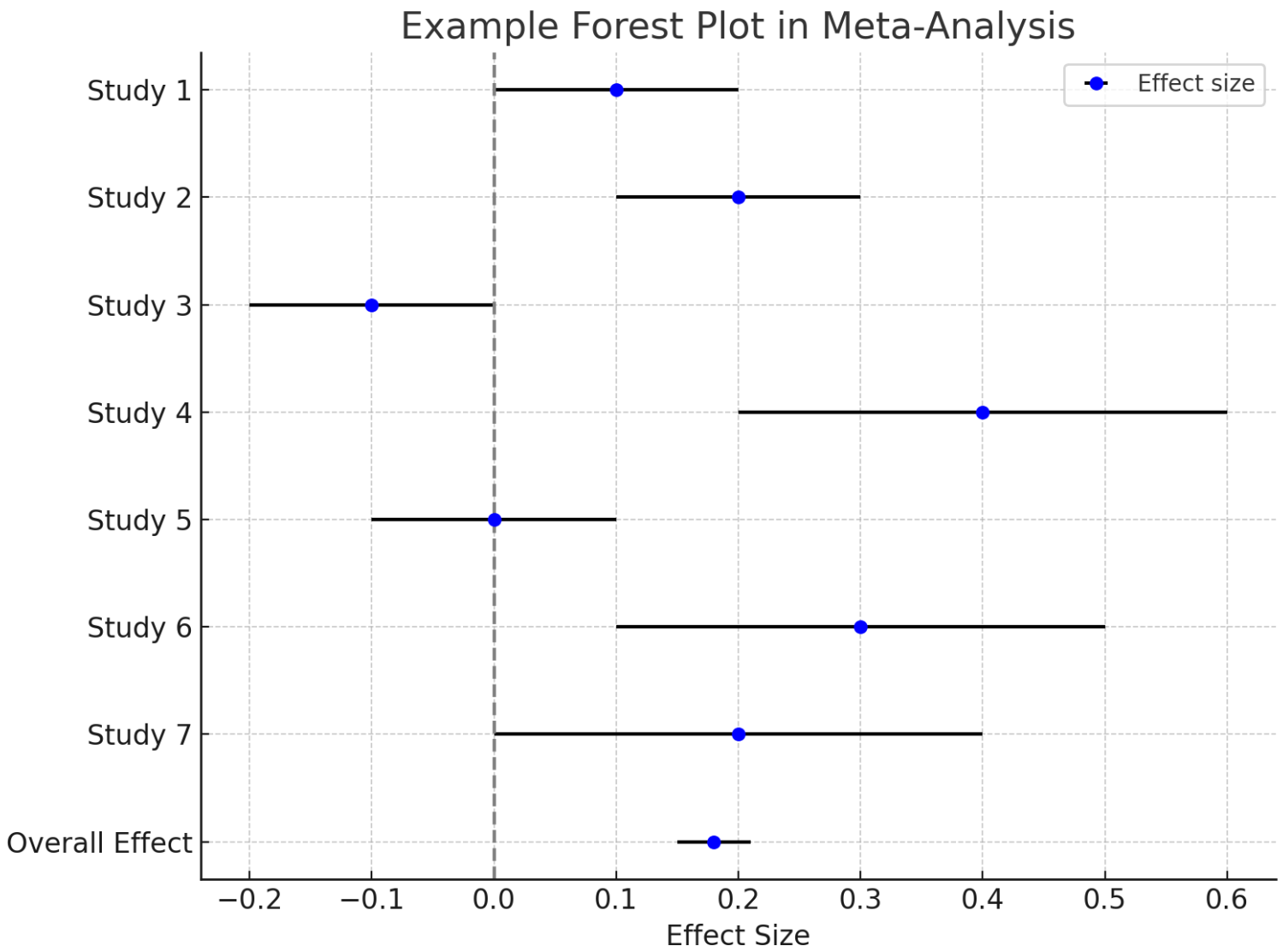
  - More accurate, default in software.

Rule of thumb: Use REML unless simplicity is needed!

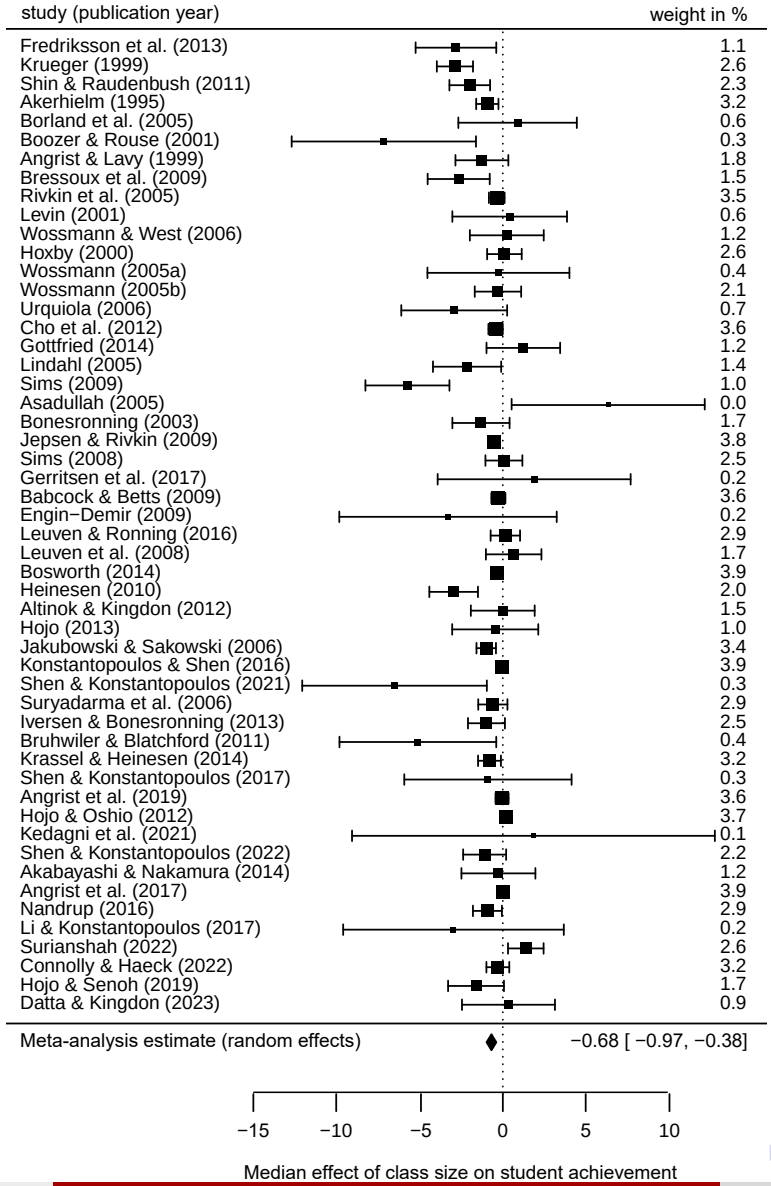
Random effects



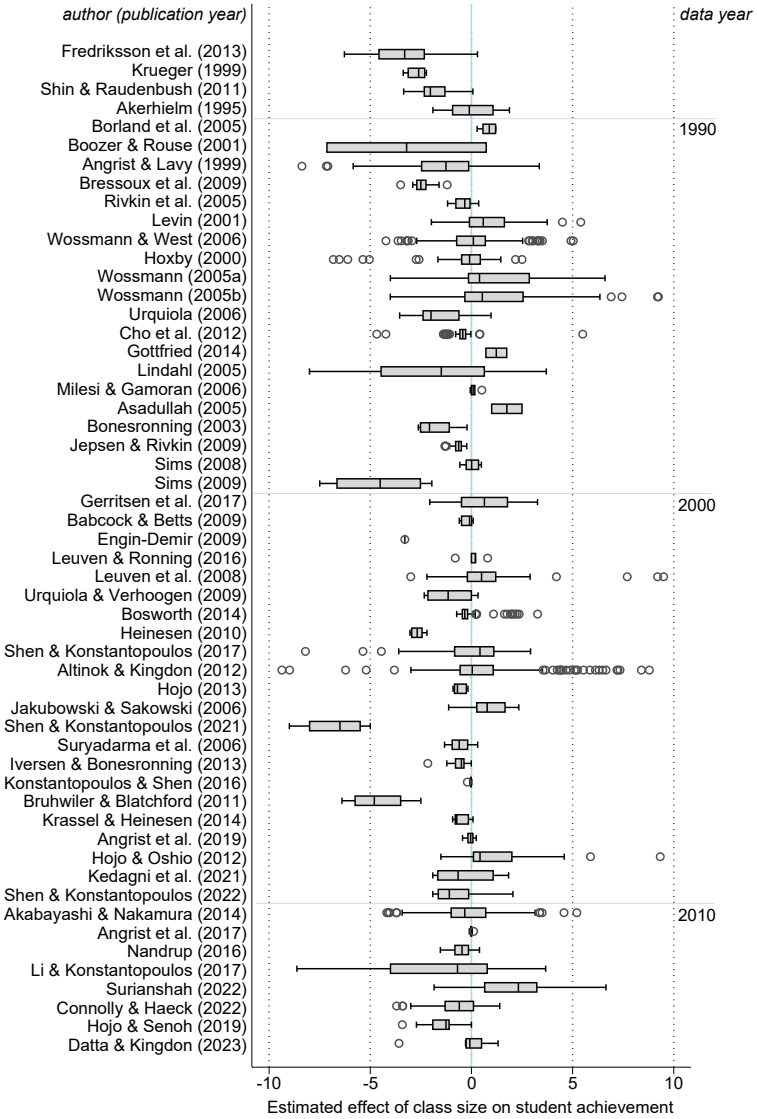
Forest plot


Example Forest Plot in Meta-Analysis
Effect size
Study 1
Study 2
Study 3
Study 4
Study 5
Study 6
Study 7
Overall Effect
−0.2
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
Effect Size

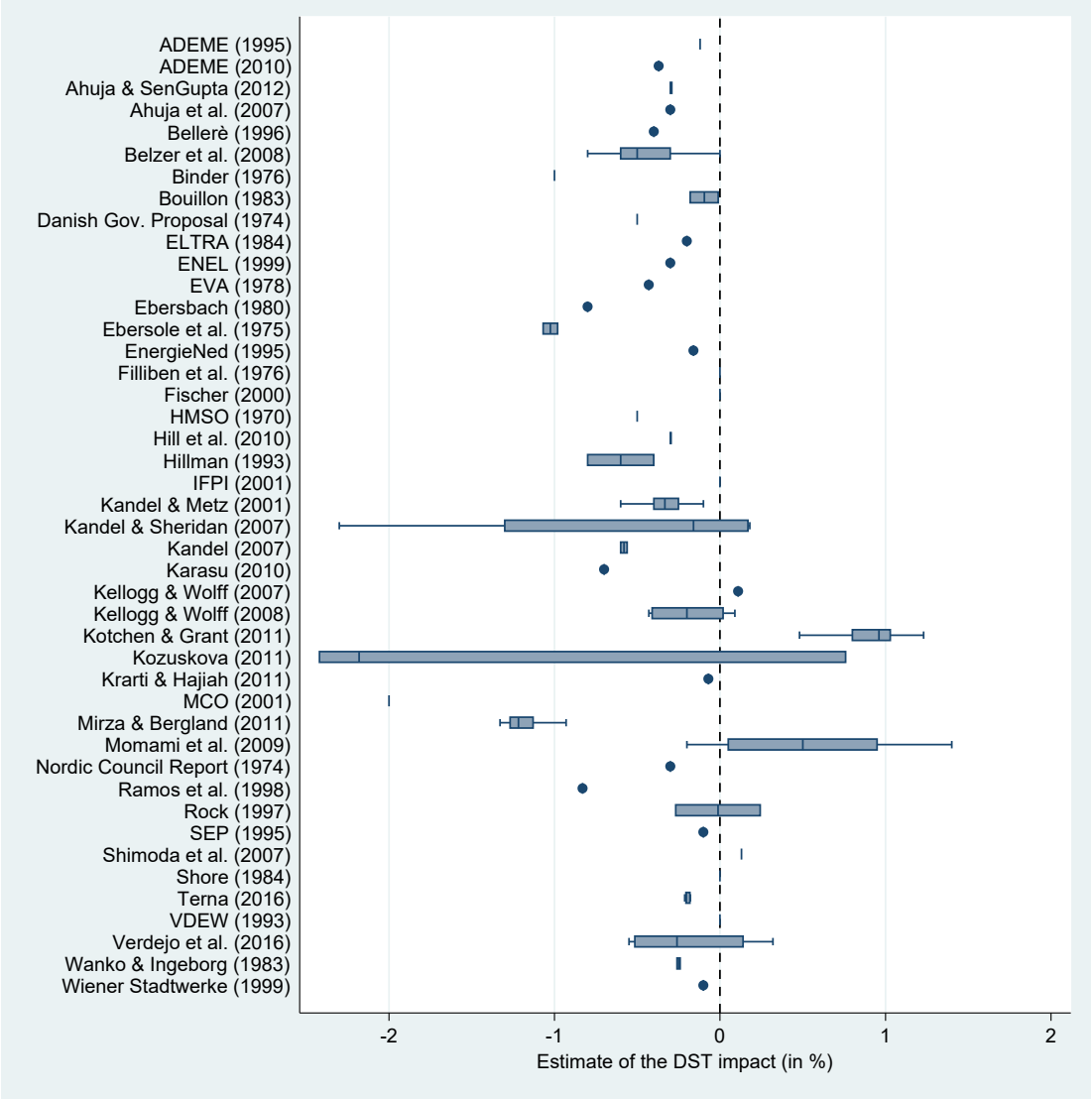
Forest plot


study (publication year)
weight in %
Fredriksson et al. (2013) 1.1
Krueger (1999) 2.6
Shin & Raudenbush (2011) 2.3
Akerhielm (1995) 3.2
Borland et al. (2005) 0.6
Boozer & Rouse (2001) 0.3
Angrist & Lavy (1999) 1.8
Bressoux et al. (2009) 1.5
Rivkin et al. (2005) 3.5
Levin (2001) 0.6
Wossmann & West (2006) 1.2
Hoxby (2000) 2.6
Wossmann (2005a) 0.4
Wossmann (2005b) 2.1
Urquiola (2006) 0.7
Cho et al. (2012) 3.6
Gottfried (2014) 1.2
Lindahl (2005) 1.4
Sims (2009) 1.0
Asadullah (2005) 0.0
Bonesronning (2003) 1.7
Jepsen & Rivkin (2009) 3.8
Sims (2008) 2.5
Gerritsen et al. (2017) 0.2
Babcock & Betts (2009) 3.6
Engin−Demir (2009) 0.2
Leuven & Ronning (2016) 2.9
Leuven et al. (2008) 1.7
Bosworth (2014) 3.9
Heinesen (2010) 2.0
Altinok & Kingdon (2012) 1.5
Hojo (2013) 1.0
Jakubowski & Sakowski (2006) 3.4
Konstantopoulos & Shen (2016) 3.9
Shen & Konstantopoulos (2021) 0.3
Suryadarma et al. (2006) 2.9
Iversen & Bonesronning (2013) 2.5
Bruhwiler & Blatchford (2011) 0.4
Krassel & Heinesen (2014) 3.2
Shen & Konstantopoulos (2017) 0.3
Angrist et al. (2019) 3.6
Hojo & Oshio (2012) 3.7
Kedagni et al. (2021) 0.1
Shen & Konstantopoulos (2022) 2.2
Akabayashi & Nakamura (2014) 1.2
Angrist et al. (2017) 3.9
Nandrup (2016) 2.9
Li & Konstantopoulos (2017) 0.2
Surianshah (2022) 2.6
Connolly & Haeck (2022) 3.2
Hojo & Senoh (2019) 1.7
Datta & Kingdon (2023) 0.9
Meta-analysis estimate (random effects)
−0.68 [−0.97, −0.38]
−15
−10
−5
0
5
10
Median effect of class size on student achievement

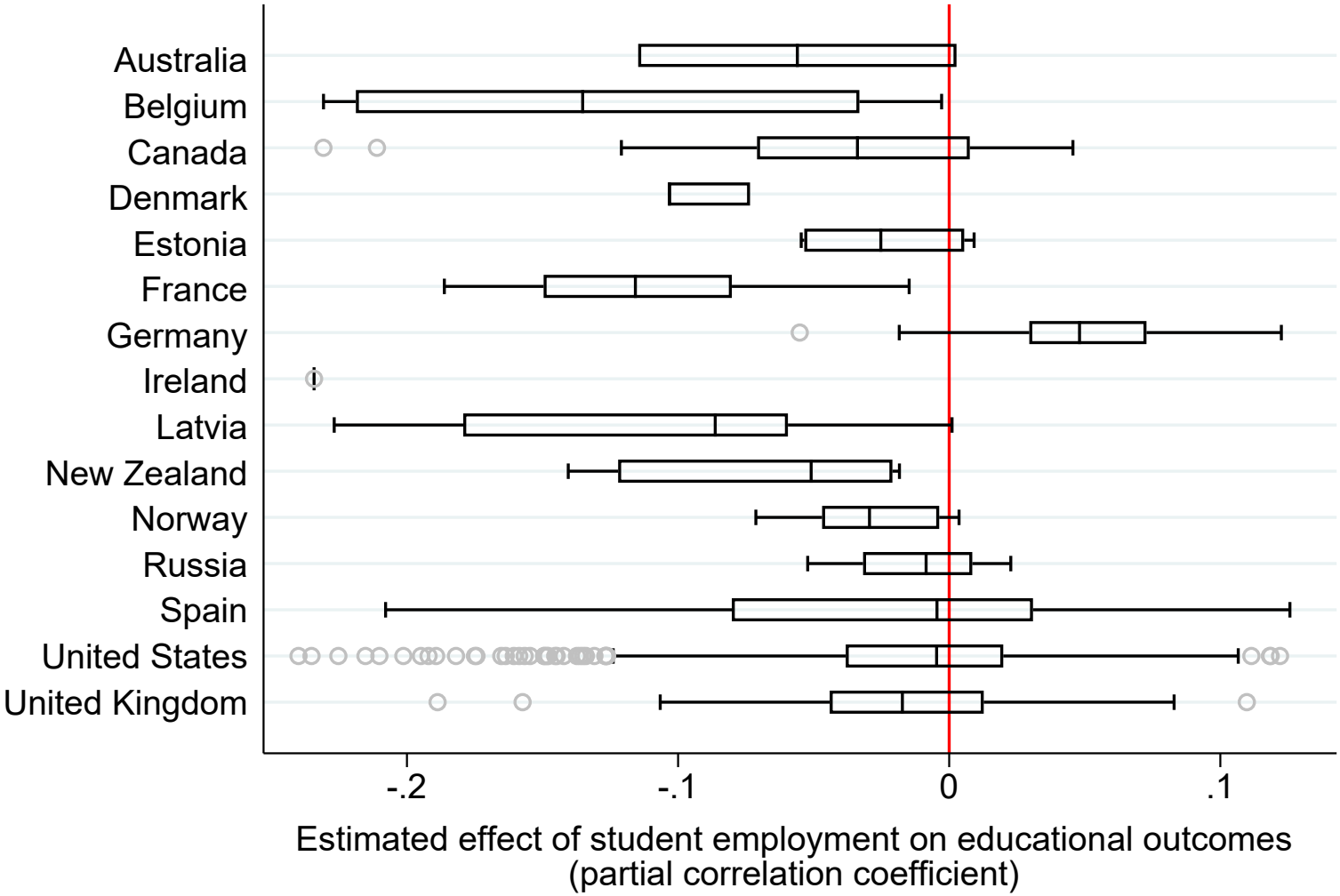
Box plot


author (publication year)
data year
Fredriksson et al. (2013)
Krueger (1999)
Shin & Raudenbush (2011)
Akerhielm (1995)
Borland et al. (2005)
Boozer & Rouse (2001)
Angrist & Lavy (1999)
Bressoux et al. (2009)
Rivkin et al. (2005)
Levin (2001)
Wossmann & West (2006)
Hoxby (2000)
Wossmann (2005a)
Wossmann (2005b)
Urquiola (2006)
Cho et al. (2012)
Gottfried (2014)
Lindahl (2005)
Milesi & Gamoran (2006)
Asadullah (2005)
Bonesronning (2003)
Jepsen & Rivkin (2009)
Sims (2008)
Sims (2009)
Gerritsen et al. (2017)
Babcock & Betts (2009)
Engin-Demir (2009)
Leuven & Ronning (2016)
Leuven et al. (2008)
Urquiola & Verhoogen (2009)
Bosworth (2014)
Heinesen (2010)
Shen & Konstantopoulos (2017)
Altinok & Kingdon (2012)
Hojo (2013)
Jakubowski & Sakowski (2006)
Shen & Konstantopoulos (2021)
Suryadarma et al. (2006)
Iversen & Bonesronning (2013)
Konstantopoulos & Shen (2016)
Bruhwiler & Blatchford (2011)
Krassel & Heinesen (2014)
Angrist et al. (2019)
Hojo & Oshio (2012)
Kedagni et al. (2021)
Shen & Konstantopoulos (2022)
Akabayashi & Nakamura (2014)
Angrist et al. (2017)
Nandrup (2016)
Li & Konstantopoulos (2017)
Surianshah (2022)
Connolly & Haeck (2022)
Hojo & Senoh (2019)
Datta & Kingdon (2023)
1990
2000
2010
-10
-5
0
5
10
Estimated effect of class size on student achievement

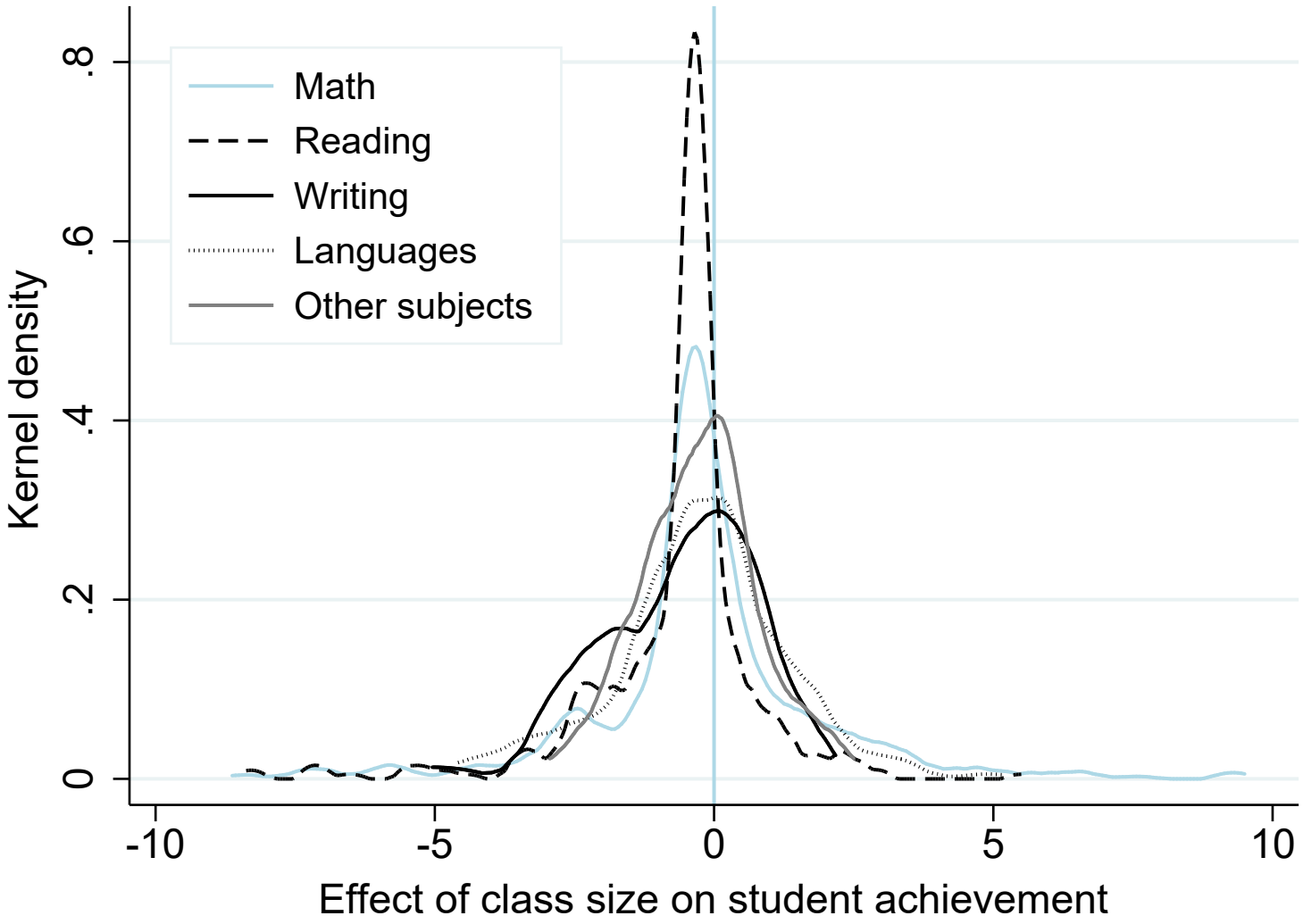
Box plot


ADEME (1995)
ADEME (2010)
Ahuja & SenGupta (2012)
Ahuja et al. (2007)
Bellerè (1996)
Belzer et al. (2008)
Binder (1976)
Bouillon (1983)
Danish Gov. Proposal (1974)
ELTRA (1984)
ENEL (1999)
EVA (1978)
Ebersbach (1980)
Ebersole et al. (1975)
EnergieNed (1995)
Filliben et al. (1976)
Fischer (2000)
HMSO (1970)
Hill et al. (2010)
Hillman (1993)
IFPI (2001)
Kandel & Metz (2001)
Kandel & Sheridan (2007)
Kandel (2007)
Karasu (2010)
Kellogg & Wolff (2007)
Kellogg & Wolff (2008)
Kotchen & Grant (2011)
Kozuskova (2011)
Krarti & Hajiah (2011)
MCO (2001)
Mirza & Bergland (2011)
Momami et al. (2009)
Nordic Council Report (1974)
Ramos et al. (1998)
Rock (1997)
SEP (1995)
Shimoda et al. (2007)
Shore (1984)
Terna (2016)
VDEW (1993)
Verdejo et al. (2016)
Wanko & Ingeborg (1983)
Wiener Stadtwerke (1999)
-2
-1
0
1
2
Estimate of the DST impact (in %)

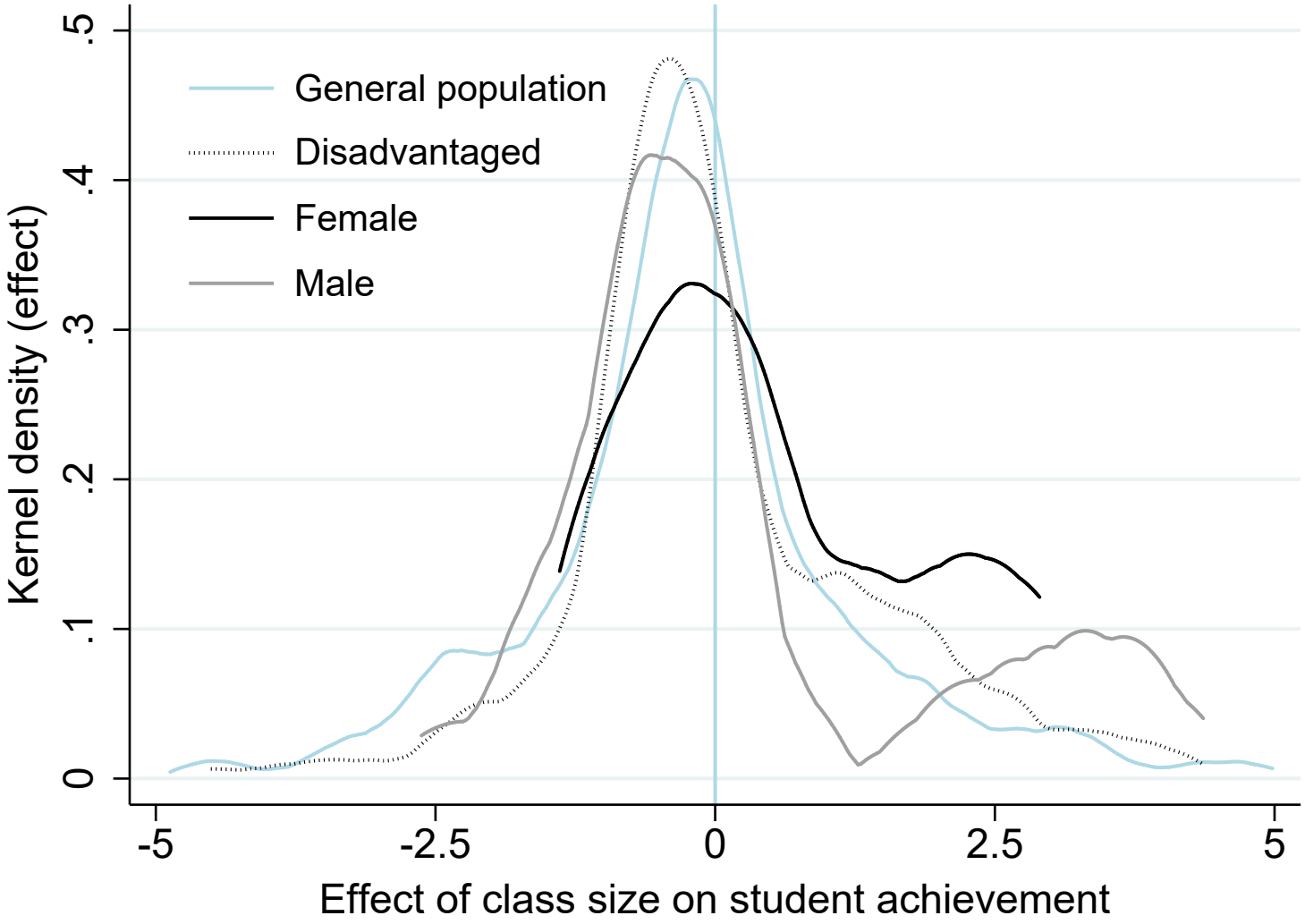
Box plot


Australia
Belgium
Canada
Denmark
Estonia
France
Germany
Ireland
Latvia
New Zealand
Norway
Russia
Spain
United States
United Kingdom
-.2
-.1
0
.1
Estimated effect of student employment on educational outcomes
(partial correlation coefficient)

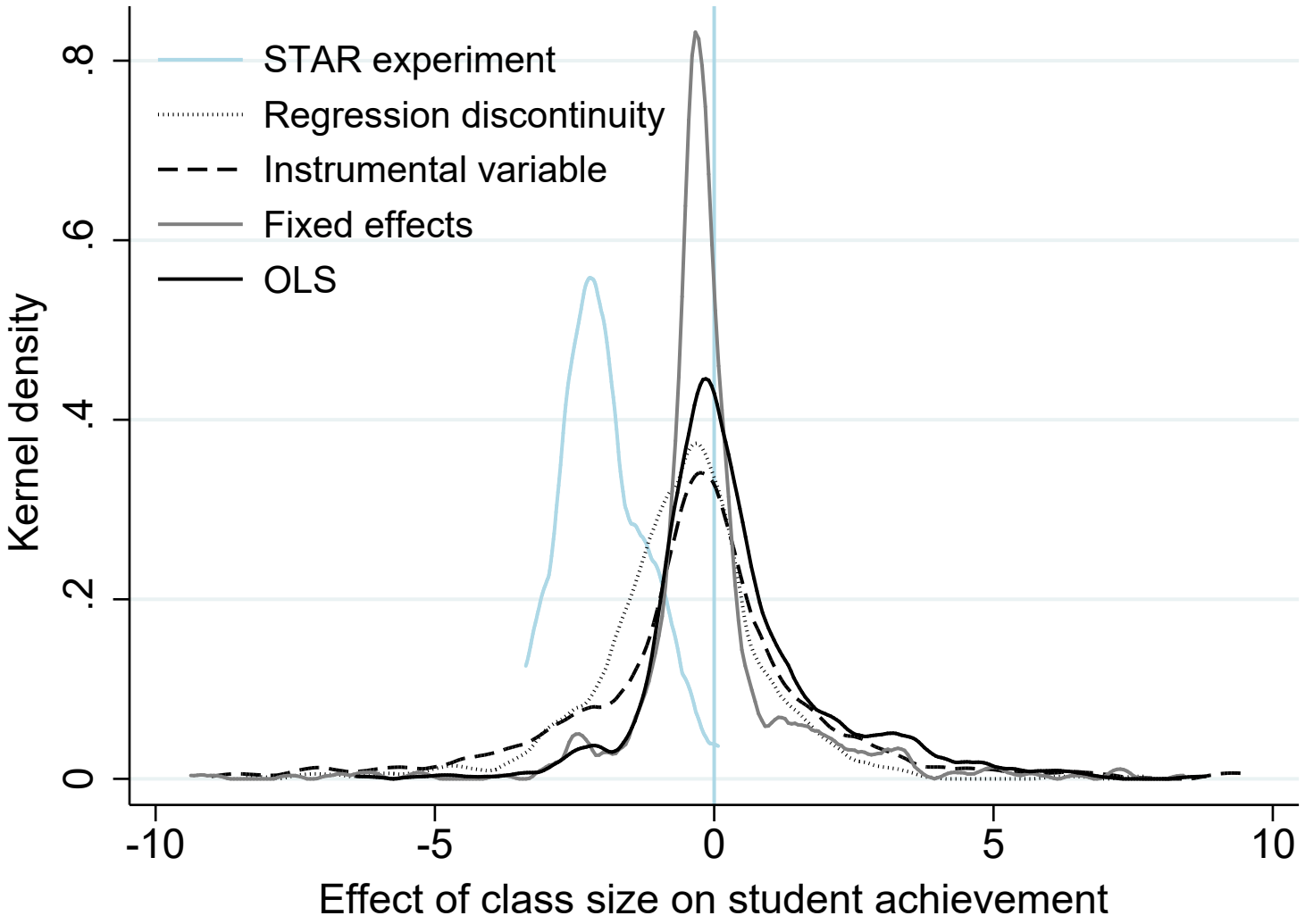
Histogram


Math
Reading
Writing
Languages
Other subjects
Kernel density
0
.2
.4
.6
.8
-10
-5
0
5
10
Effect of class size on student achievement

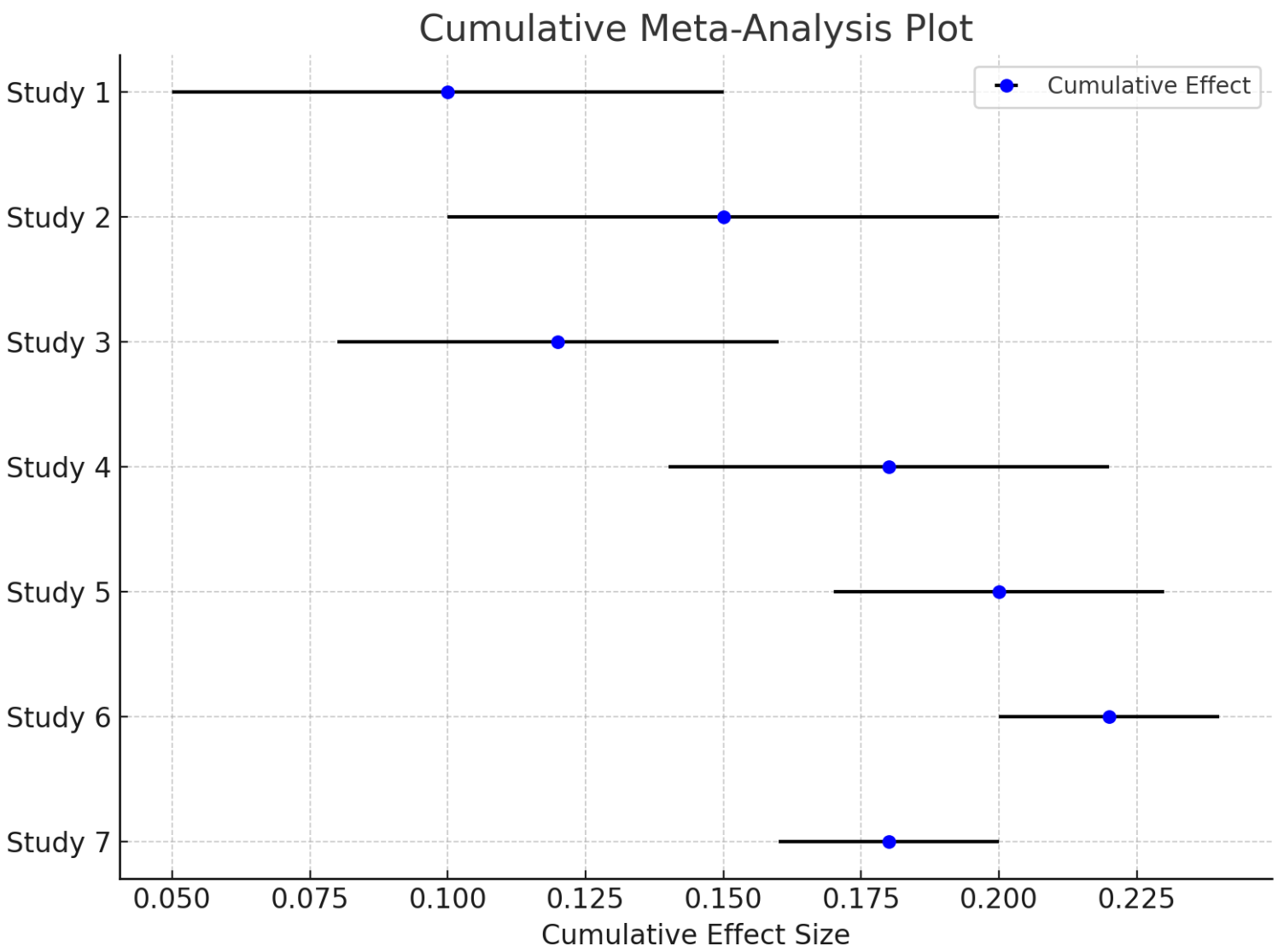
Histogram


General population
Disadvantaged
Female
Male
Kernel density (effect)
0
.1
.2
.3
.4
.5
-5
-2.5
0
2.5
5
Effect of class size on student achievement

Histogram


STAR experiment
Regression discontinuity
Instrumental variable
Fixed effects
OLS
Kernel density
0
.2
.4
.6
.8
-10
-5
0
5
10
Effect of class size on student achievement

Cumulative meta-analysis plot



Funnel plot (topic of the next lecture)